

Gas laws

1) Boyle's law

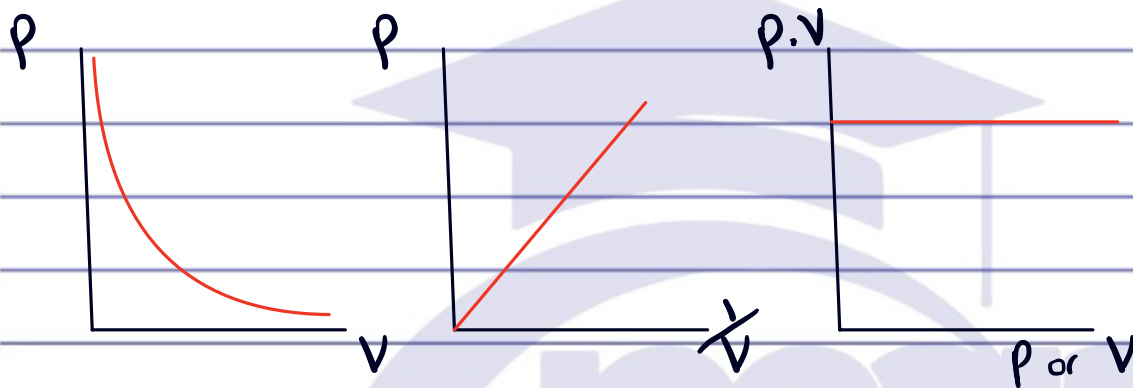
Pressure is inversely proportional to volume

$P \propto \frac{1}{V}$ keeping temperature constant. "Isothermal change."

$$P = \frac{k}{V} \quad \text{OR} \quad PV = k.$$

Formula

$$P_1 V_1 = P_2 V_2$$



Since $P \cdot V = k$ is a constant graph is a horizontal line.

2) Charles law

Volume of a gas directly proportional to its Thermodynamic Temperature.

(The term thermodynamic in Physics is used to indicate temperature measured in kelvin scale.)

$$V \propto T \quad [\text{Pressure constant}]$$

$$V = kT$$

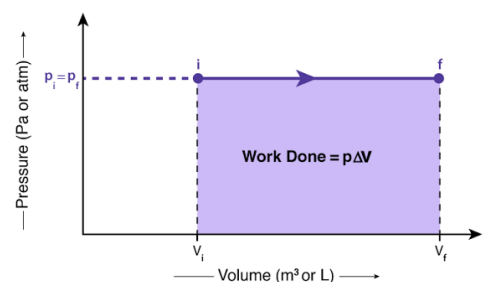
$$\frac{V}{T} = k$$

Bar: unit of Pressure.

The bar is a unit of pressure, equal to 100,000 Pa. The bar is not an SI unit, but is accepted for use with SI by BIPM - The International Bureau of Weights and Measures. The bar is convenient as it is quite close to the value of the standard atmosphere (1.01325 bar).

Isobaric Process

A thermodynamic process in which the pressure of the system remains constant ($\Delta p = 0$)



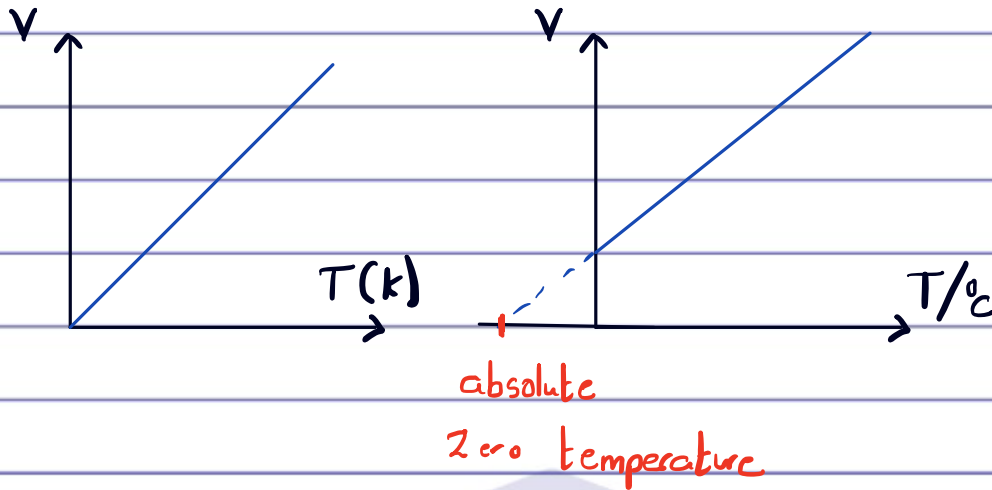
ChemistryLearner.com



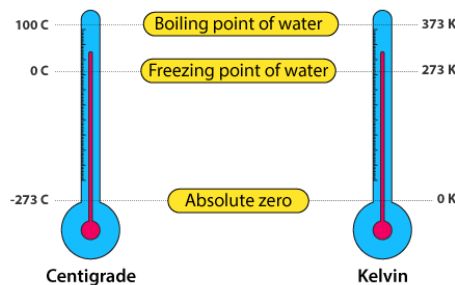
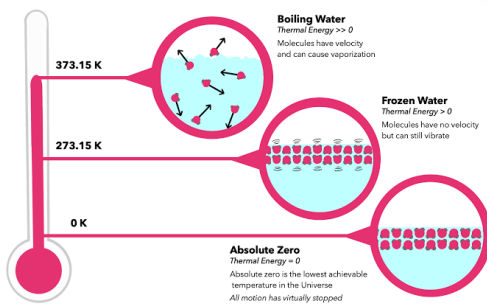
Topic: _____

Date: _____

formula =
$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$



Absolute Zero:



Convert 27°C in to kelvin.

$$27 + 273.15 = 300.15 \text{ K}$$

$$W + 273.15 = \text{Temp in Kelvin}$$

W = temperature in $^{\circ}\text{C}$.

If Examiner asks for suitable

no. of decimal places

place 0.15 in conversion

Other wise

$$W + 273 = \text{Temp in K}$$

is ok.



Properties of Thermodynamic Scale:

- ① Thermodynamic Temperature refers to temperature measured in Kelvin scale.
- ② This scale doesn't have upper fixed point.
- ③ Its lower fixed point is called absolute zero (0 K)
- ④ It is a theoretical scale doesn't depend upon any physical property.

③ Pressure law:

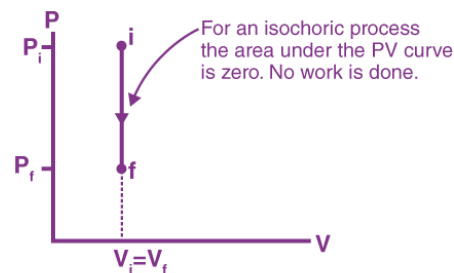
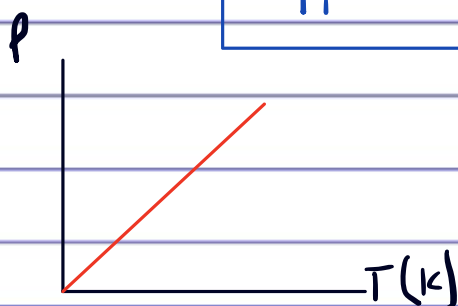
Pressure of a gas is directly proportional to its Thermodynamic temperature.

$$P \propto T \quad [\text{Volume Constant}] \quad \text{Iso Volumetric changes.}$$

$$P = kT \quad \text{or} \quad \frac{P}{T} = k$$

Formula

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$



④ Universal gas law

This law combines all 3 results into a single equation

Boyle's law $\frac{PV}{T} = k$ OR Charles law

Pressure law.

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

⑤ Ideal gas Equation:

$$PV = nRT$$

n = number of moles

R = Universal Gas constant.

$$R = 8.31 \text{ J K}^{-1} \text{ mol}^{-1}$$

Define Ideal gas: Any gas which follows the relationship $PV = nRT$ for all values of Pressure, Volume & temperature it is stated to be an Ideal gas.

"Kinetic Theory of Gases."

It refers to a set of Basic assumptions in relationship with properties movement and behavior of Ideal gas. These assumptions are as follows.

- ① Gas molecules are in continuous random motion.
- ② There are zero intermolecular forces of attraction b/w ideal gas molecules.
- ③ Volume of gas is negligible as compared to the volume of container.
- ④ Gas molecules perform perfectly Elastic collision with the walls of the container.

$$1 \text{ cm}^3 \rightarrow 10^{-6} \text{ m}^3$$

Topic: _____

Date: _____

PAGE 98

$$\text{cm}^3 \xrightarrow{\times 10^{-6}} \text{m}^3$$

MAHAD AMER

25. O/N 09/P41/Q2

An ideal gas occupies a container of volume $4.5 \times 10^3 \text{ cm}^3$ at a pressure of $2.5 \times 10^5 \text{ Pa}$ and a temperature of 290 K .

(a) Show that the number of atoms of gas in the container is 2.8×10^{23} .

$$PV = nRT$$

$$(2.5 \times 10^5) (4.5 \times 10^{-3}) = n (8.31) (290)$$

$$n = 0.4668$$

[2]

(b) Atoms of a real gas each have a diameter of $1.2 \times 10^{-10} \text{ m}$.

(i) Estimate the volume occupied by 2.8×10^{23} atoms of this gas.

volume = m^3 [2]

(ii) By reference to your answer in (i), suggest whether the real gas does approximate to an ideal gas.

.....

[2]

Avagados constant.

N_A - Avogadro's constant

MAHAD AMER

25. O/N 09/P41/Q2

An ideal gas occupies a container of volume $4.5 \times 10^3 \text{ cm}^3$ at a pressure of $2.5 \times 10^5 \text{ Pa}$ and a temperature of 290 K .

(a) Show that the number of atoms of gas in the container is 2.8×10^{23} .

$$PV = nRT$$

$$n = \frac{(2.5 \times 10^5)(4.5 \times 10^3 \times 10^{-6})}{(8.31)(290)}$$

$$n = 0.47 \quad \text{no of atoms} = n \times N_A$$

$$0.47 \times 6.02 \times 10^{23}$$

$$= 2.8 \times 10^{23}$$

[2]

(b) Atoms of a real gas each have a diameter of $1.2 \times 10^{-10} \text{ m}$.

(i) Estimate the volume occupied by 2.8×10^{23} atoms of this gas.

$$\text{Vol of each atom} = \frac{4}{3} \pi (0.6 \times 10^{-10})^3$$

$$= 9.1 \times 10^{-31}$$

$$r = 0.6 \times 10^{-10}$$

$$\text{Total volume} = 9.1 \times 10^{-31} \times 2.8 \times 10^{23}$$

$$=$$

$$\text{volume} = 2.5 \times 10^{-7} \text{ m}^3 \quad [2]$$

(ii) By reference to your answer in (i), suggest whether the real gas does approximate to an ideal gas.

$V_{\text{container}} = 4.5 \times 10^{-3}$ and $V_{\text{gas}} = 2.5 \times 10^{-7} \text{ m}^3$
 It can be said as negligible hence it can be stated as Ideal gas.

[2]

15. M/J 13/P41/Q2, M/J 13/P43/Q2

(a) State what is meant by an *ideal gas*.

.....

.....

.....

.....

[3]

(b) Two cylinders A and B are connected by a tube of negligible volume, as shown in Fig. 2.1.

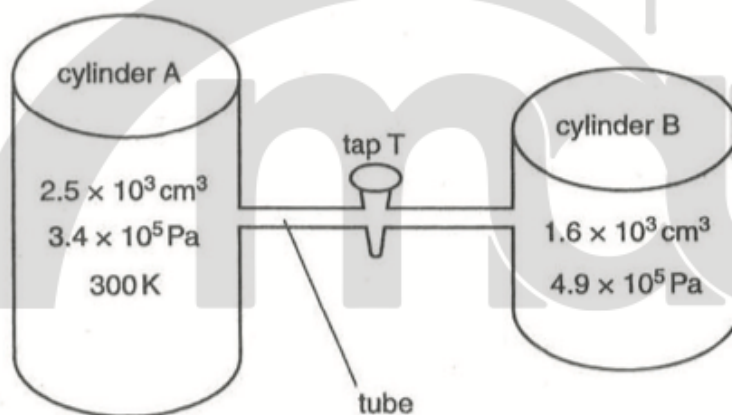


Fig. 2.1

Initially, tap T is closed. The cylinders contain an ideal gas at different pressures.

Show that cylinder A contains 0.34 mol of gas.

[1]

- (ii) Cylinder B has a constant volume of $1.6 \times 10^3 \text{ cm}^3$ and contains 0.20 mol of gas. When tap T is opened, the pressure of the gas in both cylinders is $3.9 \times 10^5 \text{ Pa}$. No thermal energy enters or leaves the gas.

Determine the final temperature of the gas.

$$PV = nRT$$

$$\begin{aligned} (4.1 \times 10^{-1}) &= (0.2 + 0.34)(3.9 \times 10^5)(T) \\ (3.9 \times 10^5) & \end{aligned}$$

temperature = K [2]

- (c) By reference to work done and change in internal energy, suggest why the temperature of the gas in cylinder A has changed.

.....

.....

.....

.....

[3]



(a) State what is meant by an *ideal gas*.

.....

.....

.....

.....

[3]

(b) Two cylinders A and B are connected by a tube of negligible volume, as shown in Fig. 2.1.

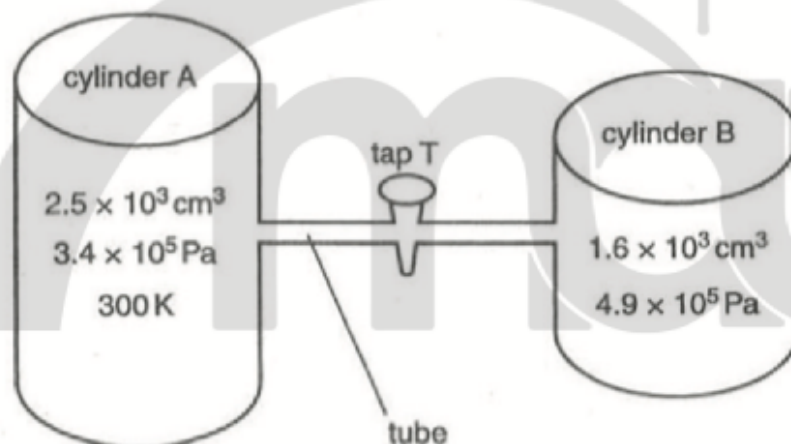


Fig. 2.1

Initially, tap T is closed. The cylinders contain an ideal gas at different pressures.

(i) Cylinder A has a constant volume of $2.5 \times 10^3 \text{ cm}^3$ and contains gas at pressure $3.4 \times 10^5 \text{ Pa}$ and temperature 300 K .

Show that cylinder A contains 0.34 mol of gas.

For A

$$PV = nRT$$

$$3.4 \times 10^5 \times 2.5 \times 10^{-3} = n$$

$$8.5 \times 300$$

$$n = 0.34 \text{ moles}$$

[1]

- (ii) Cylinder B has a constant volume of $1.6 \times 10^3 \text{ cm}^3$ and contains 0.20 mol of gas. When tap T is opened, the pressure of the gas in both cylinders is $3.9 \times 10^5 \text{ Pa}$. No thermal energy enters or leaves the gas.

Determine the final temperature of the gas.

When Tap T is opened, gas will get transferred from B to A until both systems are at the same final pressure i.e. $3.9 \times 10^5 \text{ Pa}$

For whole system:

$$p = 3.9 \times 10^5$$

$$T = ?$$

$$n = 0.34 + 0.2 = 0.54$$

$$V = (2.5 + 1.6) \times 10^{-3}$$

apply $pV = nRT$

temperature = 356K K [2]

- (c) By reference to work done and change in internal energy, suggest why the temperature of the gas in cylinder A has changed.

[3]



10. O/N 14/P41/Q3, O/N 14/P42/Q3

(a) State what is meant by an *ideal* gas.

.....
..... [1]

(b) A storage cylinder for an ideal gas has a volume of $3.0 \times 10^{-4} \text{ m}^3$. The gas is at a temperature of 23°C and a pressure of $5.0 \times 10^7 \text{ Pa}$.

(i) Show that the amount of gas in the cylinder is 6.1 mol.

[2]

(ii) The gas leaks slowly from the cylinder so that, after a time of 35 days, the pressure has reduced by 0.40%. The temperature remains constant. Calculate the average rate, in atoms per second, at which gas atoms escape from the cylinder.

rate = s^{-1} [4]

10. O/N 14/P41/Q3, O/N 14/P42/Q3

(a) State what is meant by an *ideal* gas.

.....

(b) A storage cylinder for an ideal gas has a volume of $3.0 \times 10^{-4} \text{ m}^3$. The gas is at a temperature of 23°C and a pressure of $5.0 \times 10^7 \text{ Pa}$.

(i) Show that the amount of gas in the cylinder is 6.1 mol.

$$PV = nRT$$

[2]

(ii) The gas leaks slowly from the cylinder so that, after a time of 35 days, the pressure has reduced by 0.40%. The temperature remains constant.

Calculate the average rate, in atoms per second, at which gas atoms escape from the cylinder.

According to formula $PV = nRT$ V, R, T is constant

$P \propto n$ \therefore if pressure decreases by 0.4% then no. of moles must also decrease by 0.4%. So based on this statement we can conclude

moles escaped = 0.4% of 6.1 mol

$$\frac{6.1}{100} \times 0.4$$

$$= 0.0244 \text{ mol}$$

In a period of 35 days these moles escaped

$$\# \text{ of atoms} = n \times N_A$$

$$= 0.0244 \times 6.02 \times 10^{23}$$

$$\frac{1.47 \times 10^{22}}{2} \xrightarrow{35 \times 24 \times 60 \times 60} 1$$

$$\text{rate} = \dots \text{ s}^{-1} \quad [4]$$

④ Gas molecules perform perfectly Elastic collision with the walls of the container.

As the molecules rebound elastically from the wall, they bring about a change in momentum (ΔP); thereby exerting a force on the wall & since Force per unit area leads to pressure we can say Ideal gas molecules exert Pressure on the walls of the container.

This pressure exerted by ideal gas can be calculated using the formula.

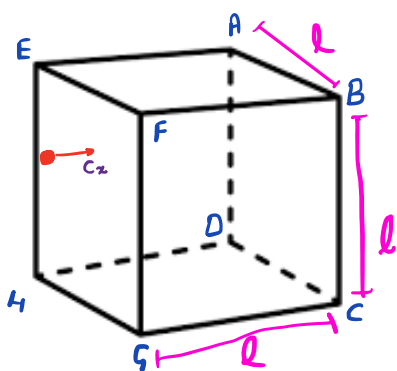
$$P = \frac{1}{3} \rho \langle c^2 \rangle \quad \text{derivation required.}$$

P = Pressure

ρ = density of ideal gas.

$\langle c^2 \rangle$ = mean square speed of ideal gas molecules

$\langle \rangle$ = mean.



The gas molecule of mass m starts from on wall (EFGH) and strikes the opposite wall (ABCD) and returns back to starting point

Assume that speed of gas molecule in x direction is " c_x "

$$F = \frac{\Delta p}{t} = \frac{mc_x - (-mc_x)}{t} = \frac{2mc_x}{t}$$

Since $t = \frac{\text{dist}}{\text{speed}} \Rightarrow \frac{2l}{c_x}$

$$F = \frac{2mc_x}{t} \Rightarrow \frac{2mc_x}{\frac{2l}{c_x}} = \frac{2m(c_x)^2}{2l}$$

$$F = \frac{m(c_x)^2}{L} \quad \longrightarrow \quad (1)$$

$$\therefore P = \frac{F}{A} \Rightarrow \frac{m(c_x)^2}{L^2} \Rightarrow \frac{m(c_x)^2}{L^3} \quad \because L^3 = V$$

$$P = \frac{m(c_x)^2}{V} \quad \longrightarrow \quad (2)$$

Assuming there are N number of molecules all travelling in x direction hence Pressure exerted becomes

$$P = \frac{N \cdot m \langle C_x^2 \rangle}{V} \quad \text{--- (3)}$$

N = No of molecules

$\langle C_x^2 \rangle$ - mean square speed in x direction

$$P = \frac{M \langle C_x^2 \rangle}{V}$$

M = mass of all molecules.

Total Mass of all molecules.

$$P = \rho \langle C_x^2 \rangle \quad \text{--- (4)}$$

$N \cdot m = M$
 \swarrow no of molecules
 \searrow mass of 1 molecule.

| |
|--|
| m = mass of one molecule N = No of molecules M = total mass of all molecules $M = Nm$ |
|--|

Last Step:

Assuming in Reality the molecules travel with almost identical speed in all directions (x direction, y direction, z direction)

$$\langle C^2 \rangle = \langle C_x^2 \rangle + \langle C_y^2 \rangle + \langle C_z^2 \rangle$$

Since $\langle C_x \rangle = \langle C_y \rangle = \langle C_z \rangle$ (Assuming identical speed in all directions)

$$\langle C^2 \rangle = 3 \langle C_x^2 \rangle \quad \text{or}$$

$$\langle C_x^2 \rangle = \frac{1}{3} \langle C^2 \rangle \quad \text{--- (5)}$$

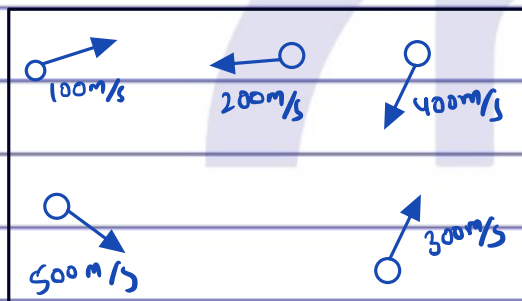
From Step 5 $\langle c_x^2 \rangle = \frac{1}{3} \langle c^2 \rangle$ let us replace this into 4th equation.

$$P = \rho \frac{1}{3} \langle c^2 \rangle$$

$$P = \frac{1}{3} \rho \langle c^2 \rangle \quad \text{Final answer.}$$

How to calculate Pressure exerted by ideal gas molecules using the above equation.

$$P = \frac{1}{3} \rho \langle c^2 \rangle$$



$$\text{density} = 4 \text{ kg m}^{-3}$$

Calculate the pressure exerted by ideal gas.

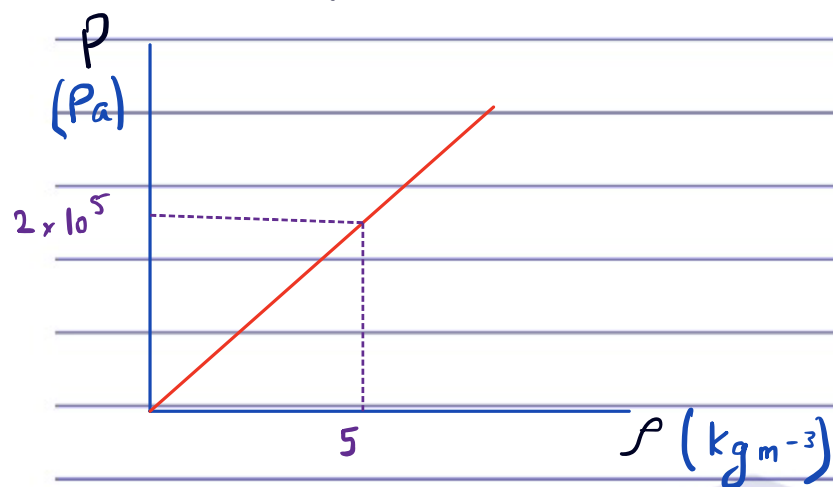
$$\langle c^2 \rangle = \frac{100^2 + 200^2 + 400^2 + 300^2 + 500^2}{5} = 1.1 \times 10^5 \text{ m}^2 \text{ s}^{-2}$$

$$\text{hence } P = \frac{1}{3} \rho \langle c^2 \rangle$$

$$P = \frac{1}{3} (4) \langle 1.1 \times 10^5 \rangle$$

$$P = 1.5 \times 10^5 \text{ Pa}$$

If You plot a graph of Pressure on Y-axis and ρ (density on X Axis)



$$P = \frac{1}{3} \langle c^2 \rangle \rho$$

$$y = mx$$

⇒ Calculate the mean square speed of the ideal gas whose graph is shown above.

mean square speed = $\langle c^2 \rangle$ = find

$$\text{grad} = \frac{1}{3} \langle c^2 \rangle$$

$$\frac{2 \times 10^5}{5} = \frac{1}{3} \langle c^2 \rangle$$

$$\langle c^2 \rangle = 120,000 \text{ m}^2/\text{s}^2$$

Question Calculate

$\sqrt{\langle c^2 \rangle}$ = To get this answer we must square root the previous answer.

$$\sqrt{120,000} = 346 \text{ m/s}$$

] This quantity is known as "root mean square speed" C_{rms}

Formula for Calculating

Derivation is Required.

① Total Kinetic Energy of all Ideal gas molecules

$$KE = \frac{1}{2} M \langle c^2 \rangle$$

Previous

$$P = \frac{1}{3} \rho \langle c^2 \rangle$$

$$P = \frac{1}{3} \frac{M}{V} \langle c^2 \rangle$$

Cross multiply

$$3VP = M \langle c^2 \rangle$$

Multiply both sides by $\frac{1}{2}$

$$\frac{3}{2} VP = \frac{1}{2} M \langle c^2 \rangle$$

$$(PV) = (nRT)$$

$$\frac{1}{2} M \langle c^2 \rangle = \frac{3}{2} PV$$

Since $PV = nRt$

$$\frac{1}{2} M \langle c^2 \rangle = \frac{3}{2} nRt$$

② Average Kinetic Energy of an Ideal gas molecules.

$$\frac{1}{2} m \langle c^2 \rangle$$

Derivation is Required.

$$\frac{1}{2} M \langle c^2 \rangle = \frac{3}{2} n RT$$

* Replace M with Nm where N is # of molecules.

$$\frac{1}{2} N m \langle c^2 \rangle = \frac{3}{2} n RT$$

$$\frac{1}{2} m \langle c^2 \rangle = \frac{3 n RT}{2 N} \quad \text{---} \quad \textcircled{1}$$

N_A (Avogadro Constant) is defined as # of molecules (N) per mole "n"

$$\text{hence } N_A = \frac{N}{n}$$

$$\frac{1}{2} m \langle c^2 \rangle = \frac{3}{2} \frac{RT}{N_A} \quad \text{---} \quad \textcircled{2}$$

Last step: Since $R = 8.31$ (Constant) and $N_A = 6.02 \times 10^{23}$ (Constant)

for simplifying we can replace $\frac{R}{N_A}$ as k

$$\text{where } k = \frac{R}{N_A}$$

$$\text{hence } \frac{1}{2} m \langle c^2 \rangle = \frac{3}{2} kT \quad \text{---} \quad \textcircled{3}$$

$$k = \frac{R}{N_A} = \frac{8.31}{6.02 \times 10^{23}} = 1.38 \times 10^{-23} \text{ J/K}$$

$$k = 1.38 \times 10^{-23}$$

k is called Boltzmann constant.

① Formula for Pressure Exerted by Ideal gas

$$P = \frac{1}{3} \rho \langle c^2 \rangle$$

$$P = \frac{1}{3} \frac{M}{V} \langle c^2 \rangle$$

$$P = \frac{1}{3} \frac{Nm}{V} \langle c^2 \rangle$$

Total KE of Ideal gas Molecules.

$$\frac{1}{3} M \langle c^2 \rangle = \frac{3}{2} P V$$

$$\frac{1}{3} M \langle c^2 \rangle = \frac{3}{2} nRT$$

Avg Kinetic Energy of Single gas molecule

$$\frac{1}{3} m \langle c^2 \rangle = \frac{3}{2} kT$$

Where $k = \frac{R}{N_A} = \frac{8.31}{6.02 \times 10^{23}} = k = 1.38 \times 10^{-23}$

Boltzmann constant.

④ Based on

$$\frac{1}{2} m \langle c^2 \rangle = \frac{3}{2} k T$$

Since m and k are both constants.

KE of ideal gas only depends upon its temperature.

$$\langle c^2 \rangle \propto T$$

OR

$$\sqrt{\langle c^2 \rangle} \propto \sqrt{T}$$

$$c_{rms} \propto \sqrt{T}$$

⑤ Q :- Use the above result to prove that for an ideal gas, the product of its pressure & Volume can be expressed as follows.

$$P \cdot V = N k T \quad (\text{Derivation is Required})$$

$$P = \frac{1}{3} \rho \langle c^2 \rangle \quad \text{and} \quad \frac{1}{2} m \langle c^2 \rangle = \frac{3}{2} k T$$

$$P = \frac{1}{3} \frac{N m}{V} \langle c^2 \rangle$$

$$3 P V = N m \langle c^2 \rangle \quad \text{multiply by } \frac{1}{2}$$

$$\frac{3}{2} P V = \frac{1}{2} N \cdot m \langle c^2 \rangle$$

$$\therefore \frac{3}{2} P V = \frac{3}{2} k T \cdot N$$

$$P V = N k T$$

27. M/J 08/P4/Q2

(a) Explain qualitatively how molecular movement causes the pressure exerted by a gas.

Molecules collide they cause change in momentum
which leads to force and Force per unit volume
is P

[3]

(b) The density of neon gas at a temperature of 273K and a pressure of 1.02×10^5 Pa is 0.900 kg m^{-3} . Neon may be assumed to be an ideal gas.

Calculate the root-mean-square (r.m.s.) speed of neon atoms at

(i) 273K,

speed = ms^{-1} [3]

(ii) 546K,

speed = ms^{-1} [2]



- (c) The calculations in (b) are based on the density for neon being 0.900 kg m^{-3} . Suggest the effect, if any, on the root-mean-square speed of changing the density at constant temperature.

[2]



(a) Explain qualitatively how molecular movement causes the pressure exerted by a gas.

- Molecules collide elastically to cause Δp (change in momentum). $\Delta p \div t$ causes force to be exerted.

* Force per unit Area gives rise to pressure.

[3]

(b) The density of neon gas at a temperature of 273K and a pressure of 1.02×10^5 Pa is 0.900 kg m^{-3} . Neon may be assumed to be an ideal gas.

$$P = \frac{1}{3} \rho \langle c^2 \rangle$$

$$1.02 \times 10^5 = \frac{1}{3} (0.9) \langle c^2 \rangle$$

$$\langle c^2 \rangle = 340,000 \text{ m}^2 \text{ s}^{-2}$$

$$\sqrt{340,000} = 580$$

speed = 580 ms^{-1} [3]

(ii) 546K.

$$C_{\text{rms}} \propto \sqrt{T}$$

$$C_{\text{rms}} = \frac{580}{\sqrt{273}} (\sqrt{546})$$

$$C_{\text{rms}} = w \sqrt{T}$$

$$= 820 \text{ ms}^{-1}$$

$$\frac{580}{\sqrt{273}} = w$$

speed = 820 ms^{-1} [2]

(c) The calculations in (b) are based on the density for neon being 0.900 kg m^{-3} . Suggest the effect, if any, on the root-mean-square speed of changing the density at constant temperature.

As $C_{rms} \propto \sqrt{T}$, if T remains constant C_{rms} also remains unchanged.

[2]



- (a) Some gas, initially at a temperature of 27.2°C , is heated so that its temperature rises to 38.8°C .

Calculate, in kelvin, to an appropriate number of decimal places,

- (i) the initial temperature of the gas,

initial temperature = K [2]

- (ii) the rise in temperature.

rise in temperature = K [1]

- (b) The pressure p of an ideal gas is given by the expression

$$p = \frac{1}{3} \rho \langle c^2 \rangle$$

where ρ is the density of the gas.

- (i) State the meaning of the symbol $\langle c^2 \rangle$.

.....
 [1]

- (ii) Use the expression to show that the mean kinetic energy $\langle E_K \rangle$ of the atoms of an ideal gas is given by the expression

$$\langle E_K \rangle = \frac{3}{2} kT.$$

Explain any symbols that you use.

.....

 [4]

- (c) Helium-4 may be assumed to behave as an ideal gas.
A cylinder has a constant volume of $7.8 \times 10^3 \text{ cm}^3$ and contains helium-4 gas at a pressure of $2.1 \times 10^7 \text{ Pa}$ and at a temperature of 290 K .

Calculate, for the helium gas,

- (i) the amount of gas,

amount = mol [2]

- (ii) the mean kinetic energy of the atoms,

mean kinetic energy = J [2]

- (iii) the total internal energy.

internal energy = J [3]

24. M/J 10/P41/Q2

- (a) Some gas, initially at a temperature of 27.2°C , is heated so that its temperature rises to 38.8°C .

Calculate, in kelvin, to an appropriate number of decimal places,

- (i) the initial temperature of the gas,

$27.2 + 273.15$] Appropriate no of D.P

$27.2 + 273.2$

initial temperature = 300.4 K [2]

- (ii) the rise in temperature.

27.2 to 38.3

rise $^{\circ}\text{C}$ = rise (K)

rise in temperature = 11.6 K [1]

- (b) The pressure p of an ideal gas is given by the expression

$$p = \frac{1}{3} \rho \langle c^2 \rangle$$

where ρ is the density of the gas.

- (i) State the meaning of the symbol $\langle c^2 \rangle$.

mean square speed

[1]

- (ii) Use the expression to show that the mean kinetic energy $\langle E_K \rangle$ of the atoms of an ideal gas is given by the expression

$$\langle E_K \rangle = \frac{3}{2} kT.$$

Explain any symbols that you use.

$$p = \frac{1}{3} \rho \langle c^2 \rangle$$

$$p = \frac{1}{3} \frac{Nm}{V} \langle c^2 \rangle$$

$$3 pV = Nm \langle c^2 \rangle$$

$$\frac{3}{2} pV = \frac{Nm}{2} \langle c^2 \rangle$$

$$\frac{3}{2} nRT = \frac{Nm}{2} \langle c^2 \rangle$$

$$\frac{3nRT}{2N} = \frac{1}{2} m \langle c^2 \rangle$$

$$\frac{3}{2} \left(\frac{RT}{N_A} \right) = kE$$

$$\therefore k = \frac{R}{N_A}$$

[4]

N_A = Avogadro's constant

$$\frac{3}{2} kT = kE$$

(c) Helium-4 may be assumed to behave as an ideal gas.

A cylinder has a constant volume of $7.8 \times 10^3 \text{ cm}^3$ and contains helium-4 gas at a pressure of $2.1 \times 10^7 \text{ Pa}$ and at a temperature of 290 K .

Calculate, for the helium gas,

(i) the amount of gas,

$$pV = nRT$$
$$\frac{2.1 \times 10^7 (7.8 \times 10^{-3})}{8.31 \times 290} =$$

amount = 68 mol mol [2]

(ii) the mean kinetic energy of the atoms,

$$KE = \frac{3}{2} kT$$
$$= \frac{3}{2} (1.38 \times 10^{-23}) (290)$$

mean kinetic energy = 6×10^{-21} J [2]

(iii) the total internal energy = Total KE

$$6 \times 10^{-21} \times 68 \times 6.02 \times 10^{23} =$$

internal energy = 2.5×10^5 J [3]

Internal Energy:

Internal Energy is the sum of KE and Potential Energy (Elastic Potential Energy) of the molecules

- * KE depends on temperature
- * Elastic Potential Energy depends upon Intermolecular bonds.

* For Ideal gas we know that there are no intermolecular forces of attraction hence

$$PE = 0$$

Internal Energy = Kinetic Energy.

(Only for Ideal gas)

- (a) One assumption of the kinetic theory of gases is that gas molecules behave as if they are hard, elastic identical spheres.

State two other assumptions of the kinetic theory of gases.

1. Vol^d gas is negligible as compare to volume of container.
2. Gas molecules are in continuous random motion.

[2]

- (b) Using the kinetic theory of gases, it can be shown that the product of the pressure p and the volume V of an ideal gas is given by the expression

$$pV = \frac{1}{3}Nm\langle c^2 \rangle \rightarrow p = \frac{1}{3}\rho\langle c^2 \rangle$$

$$p = \frac{1}{3} \frac{N \cdot m}{V} \langle c^2 \rangle$$

where m is the mass of a gas molecule.

- (i) State the meaning of the symbol

1. N ,
No of molecules [1]
2. $\langle c^2 \rangle$,
mean square speed [1]

- (ii) Use the expression to deduce that the mean kinetic energy $\langle E_k \rangle$ of a gas molecule at temperature T is given by the equation

$$\langle E_k \rangle = \frac{3}{2} kT$$

where k is a constant.

$$\begin{aligned}
 p &= \frac{1}{3} \rho \langle c^2 \rangle \\
 p &= \frac{1}{3} \frac{Nm}{V} \langle c^2 \rangle \\
 3 pV &= Nm \langle c^2 \rangle \\
 \frac{3}{2} pV &= \frac{Nm}{2} \langle c^2 \rangle
 \end{aligned}
 \quad \left| \quad
 \begin{aligned}
 \frac{3}{2} nRT &= \frac{Nm}{2} \langle c^2 \rangle \\
 \frac{3 nRT}{2N} &= \frac{1}{2} m \langle c^2 \rangle \\
 \frac{3}{2} \left(\frac{RT}{N_A} \right) &= KE
 \end{aligned}
 \right.
 \quad \cdot \quad k = \frac{R}{N_A} \quad [2]$$

- (c) (i) State what is meant by the *internal energy* of a substance.

.....

.....

.....

[2]

- (ii) Use the equation in (b)(ii) to explain that, for an ideal gas, a change in internal energy ΔU is given by

$$\Delta U \propto \Delta T$$

where ΔT is the change in temperature of the gas.

.....

.....

.....

[2]

- (b) A cube of volume V contains N molecules of an ideal gas. Each molecule has a component c_x of velocity normal to one side S of the cube, as shown in Fig. 2.1.

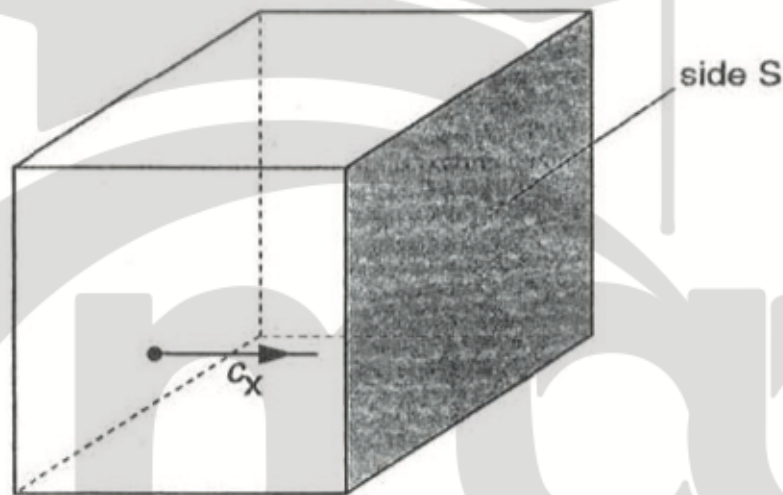


Fig. 2.1

The pressure p of the gas due to the component c_x of velocity is given by the expression

$$pV = Nmc_x^2 \quad \text{Step 1}$$

where m is the mass of a molecule.

Explain how the expression leads to the relation

$$pV = \frac{1}{3}Nm\langle c^2 \rangle \quad \text{Step 2}$$

where $\langle c^2 \rangle$ is the mean square speed of the molecules.

Assuming that molecules travel with identical speeds in x , y and z

$$\langle c^2 \rangle = \langle c_x^2 \rangle + \langle c_y^2 \rangle + \langle c_z^2 \rangle$$

\therefore Since $\langle c_x^2 \rangle = \langle c_y^2 \rangle = \langle c_z^2 \rangle$

$$\langle c^2 \rangle = 3 \langle c_x^2 \rangle \Rightarrow \langle c_x^2 \rangle = \frac{1}{3} \langle c^2 \rangle$$

Replace $\langle c_x^2 \rangle = \frac{1}{3} \langle c^2 \rangle$ to get $pV = \frac{1}{3}Nm\langle c^2 \rangle$

[3]

(c) The molecules of an ideal gas have a root-mean-square (r.m.s.) speed of 520ms^{-1} at a temperature of 27°C .

Calculate the r.m.s. speed of the molecules at a temperature of 100°C .

$27 + 273 = 300\text{K}$

$C_{\text{rms}} = 520 \text{ at } 300\text{K}$



r.m.s. speed = ms^{-1} [3]



Topic: _____

Date: _____



Topic: _____

Date: _____



Topic: _____

Date: _____



Topic: _____

Date: _____



Topic: _____

Date: _____



Topic: _____

Date: _____



Topic: _____

Date: _____



Topic: _____

Date: _____



Topic: _____

Date: _____



Topic: _____

Date: _____



Topic: _____

Date: _____



Topic: _____

Date: _____



Topic: _____

Date: _____



Topic: _____

Date: _____



Topic: _____

Date: _____

