Paper 5

A Level Physics

Planning, Analysis and Evaluation
Complete Solution

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To all those people who are not afraid of doing things no one has ever done before.

CONTENTS

INTRODUCTION

Paper 5 consists of two questions (each carrying 15 marks) and the time duration of this paper is 1 hour and 15 minutes.

The examination paper does not require laboratory facilities.

The first question is the planning question, in which candidates are required to design an experimental investigation of a given problem, and answer the question with a labelled diagram and an extended piece of writing.

The second question is the analysis, conclusions and evaluation question, in which candidates are given an equation and some experimental data. From these they are required to find the values for different constants. They are also required to estimate the uncertainties in their answers.

Some questions on this paper may be set in areas of Physics that are difficult to investigate experimentally in school laboratories, but no question requires prior knowledge of theory or equipment that is beyond the syllabus: candidates are given all the information that they need.

Note: Average score in this paper to secure A grade is **21** (out of 30).

Question 1: Planning

Generic Mark Scheme

(Before October/November 2015)

Breakdown of skills	Mark allocation
Defining the problem	3 marks
Methods of data collection	5 marks
Method of analysis	2 marks
Safety considerations	1 mark
Additional detail	4 marks

(After October/November 2015)

Breakdown of skills	Mark allocation
Defining the problem	2 marks
Methods of data collection	4 marks
Method of analysis	3 marks
Additional detail including safety considerations	6 marks

Note:

In the mark scheme of question 1:

- P denotes problem-defining mark
- M denotes method-of-data-collection mark
- A denotes method-of-analysis mark
- S denotes safety-consideration mark
- D denotes additional-detail mark

In order to understand the following rules better, first go through sample question 1.1 and its solution (on page 6).

1.1 Rules for Defining the Problem (P-marks)

1 The independent variable should be the one that can be varied easily, and the dependant variable should be the one that varies by itself as the independent variable is varied (i.e. it does not need to be varied separately).

2 The independent and dependent variables should both be there in the relationship given in the question.

3 All those variables (physical quantities) upon which the dependent and/or independent variables depend, directly or indirectly, should be kept constant, because: when these variables vary, the dependent and/or independent variables also vary with them. These variables may or may not be there in the given relationship.

Note: Sometimes, any one of the two quantities in the given relationship may be chosen as the independent variable, and the other quantity, as the dependent variable.

1.2 Rules for Method of Data Collection (M-marks)

1 A clear labelled diagram, illustrating the **assembled** experimental setup, should be drawn in the space provided.

Note: Sometimes (e.g. in the experiments of electricity), a separate circuit diagram containing conventional symbols of the circuit components should also be drawn.

2 Method of varying the independent variable should be described if appropriate.

3 Methods of determining the dependent and independent variables should be described, and the names of the measuring instruments to be used should also be stated explicitly.

Note: The measuring instrument chosen should be appropriate; that is, it should be able to measure the required physical quantity with appropriate accuracy (i.e. with reasonably small percentage uncertainty).

4 Method of minimising the environmental effect (e.g. background reading) on the readings to be taken should be described if appropriate.

5 Methods of keeping other variables constant should be described if appropriate.

For full marks to be scored in this section, the overall arrangement should be workable; that is, it should be possible to collect the required data without undue difficulty if the apparatus is assembled as described.

1.3 Rules for Describing Method of Analysis (A-marks)

1 From the equation given in the question, another equation should be written (either by simply rearranging the given equation or by using logarithmic identities) so that a straight-line graph may be obtained between (the functions of) dependent and independent variables.

2 The functions of dependent and independent variables, to be taken along the *y*- and *x*-axes respectively, should be identified.

3 The expressions for the gradient and *y*-intercept of the graph should be identified.

4 If the experiment is designed to test a relationship, then a statement of the following form should be given:

"If the graph turns out to be straight-line and passes/does not pass through the origin, then the suggested relationship is correct".

Note: Sometimes a statement like: "If the graph turns out to be straight-line, then the suggested relationship is correct" may also serve the purpose.

5 If the experiment is designed to determine a value for some constant, then the constant **must** be made the subject of the equation (from the expression of gradient or *y*-intercept).

1.4 Rule for Safety Considerations (S-mark)

Both, the risk of the experiment and the safety precaution to be taken to minimize the risk, should be stated.

1.5 Additional Detail Marks (D-marks)

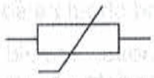
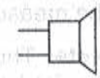
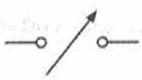
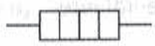
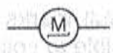
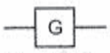
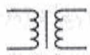
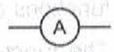
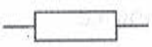
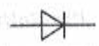
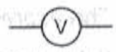
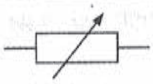
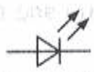
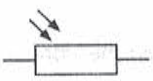
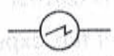
These marks may be awarded for:

- identifying which additional variable is to be kept constant;
- describing method how additional variable is to be kept constant;
- describing method to reduce random and/or systematic errors in the measurements to be taken (i.e. to increase the precision and/or accuracy in the results);
- drawing a separate diagram of a circuit needed to make a particular measurement;
- identifying an additional risk of the experiment and stating a safety precaution to minimise it.

Note: A candidate can score only **four** D-marks at the most.

1.6 Some Important Electrical Components and Their Circuit Symbols

The table below shows some electrical components and their circuit symbols that are important from the examination point of view.

Component	Symbol	Component	Symbol	Component	Symbol
power supply		thermistor		loudspeaker	
variable power supply		heater		microphone	
a.c. power supply		Inductor (or coil)		motor	
signal generator		transformer		galvanometer	
junction of conductors		lamp		ammeter	
fixed resistor		diode		voltmeter	
variable resistor (or rheostat)		light-emitting diode (LED)		ohmmeter	
light-dependant resistor (LDR)		electric bell		oscilloscope (c.r.o.)	

Advantage of Variable D.C. Power Supply over Simple D.C. Power Supply

- The output voltage of variable power supply, and therefore the current in the circuit, can be controlled by adjusting its voltage knob; hence no need for a separate variable resistor (or rheostat) in the circuit to control current.

Advantages of Signal Generator over A.C. Power Supply

- The frequency of output voltage of signal generator can be increased or decreased to produce noticeable effects (e.g. in the experiments involving electromagnetic induction).
- The output (peak or rms) voltage of signal generator, and therefore the current in the circuit, can be controlled by adjusting its voltage knob; hence no need for a separate variable resistor (or rheostat) in the circuit to control current.

Advantage of Storage C.R.O. over Voltmeter

- A storage c.r.o. can record a rapidly changing input voltage signal, and then display it on its screen as a still trace (i.e. voltage vs. time graph).

Note: If a component is not represented by its conventional symbol in the circuit diagram, then it must be labelled.

Sample Question 1.1 *(P05/M/J/07)*

It is useful to know how the speed of an object is affected by its size when it moves through liquid in a confined space. In a laboratory this can be modelled by dropping small steel balls through oil. It is suggested that the terminal velocity v is related to the radius r of a steel ball by the equation:

$$v = kr^2$$

where k is a constant. Design a laboratory experiment to investigate whether v is related to r as indicated in the above equation. You should draw a diagram showing the arrangement of your equipment. In your account you should pay particular attention to:

(a) the procedure to be followed,

(b) how the radius of the steel ball would be measured,

(c) how the terminal velocity of the steel ball in oil would be measured,

(d) the control of variables,

(e) how the data would be analysed,

(f) any safety precautions that you would take. [15]

Conventions within the Account

BRACKETS

Where brackets are shown in the account, **not** in the diagram, the candidate is not required to give the bracketed information in order to earn the available marks. Text written inside the brackets makes only the part of explanation not required.

BLUE TEXT

In the account, blue text makes the part of information that has already been indicated in the diagram; so the candidate is not required to give the blue-text information again in order to earn the available marks.

UNDERLINING

In the account, underlining indicates information that is essential for marks to be awarded.

Solution

In this experiment, I will:

- vary r and determine v (for each value of r). [P + P]
  (i.e. r is the independent variable, and v is the dependent variable.)
- keep temperature (of the oil) constant. [P]
  (so that the density of the oil upon which the drag force depends remains constant.)

To collect and analyse the data, I will take the following steps:

1 Build the experimental setup as shown in Fig. 1.1.1. In the experimental setup:

- the purpose of using the long tube of oil, having two marks on it, is to measure the time taken for the ball to fall from one mark to the other when dropped through the oil. [M]
- the purpose of using the clear oil is to view the ball, moving through the oil, with ease. [D]

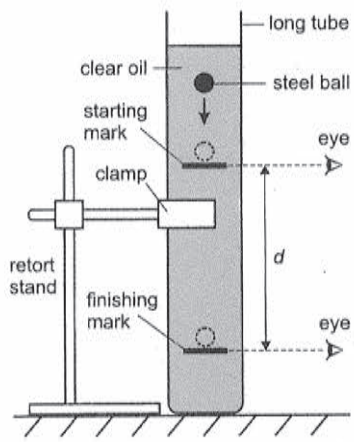


Figure 1.1.1

- the purpose of using the retort stand and clamp arrangement is to hold the long tube in the upright position. [D]

When building the experimental setup, I will:

- draw the starting mark well below the oil surface to make sure that the ball has attained terminal velocity before it reaches the mark. [M]
- keep starting and finishing marks as far apart as possible so that the time t taken for the ball to fall through the distance d is (reasonably) large, and hence the percentage uncertainty in its measurement is (reasonably) small. [D + D]
- use a spirit level to make sure that the long tube is vertical. [D]

2 Measure distance d between the starting and finishing marks with a metre rule.

3 Take steel balls of different diameters.

4 Wash and dry them. [D]

5 Measure the diameter of a steel ball with a micrometer screw gauge. [M]

When measuring the diameter, I will take multiple readings in different directions and find the average value. [D]

6 Determine the radius r of the ball by dividing its average diameter by 2.

7 Drop the ball near the oil surface.

8 With a stopwatch, measure the time t taken for the ball to fall from the starting mark to the finishing mark. [M]

When measuring the time t, I will avoid parallax error by keeping the eye at right level as shown in Fig. 1.1.1. [D]

9 Retrieve the ball using a magnet. [D]

10 Repeat steps **7** and **8** for the same ball and find the average value of t. [D]

11 Determine the terminal velocity v of the ball using the formula:

$$v = \frac{d}{t}$$ [M]

12 Take another steel ball of different diameter.

13 Repeat the procedure from step **5** to **11**, and thus obtain about 6 sets of results.

14 From the equation given in the question:

$$v = kr^2$$

it follows that the gradient of v vs. r^2 graph is equal to k. [D]

15 Plot a graph of v against r^2. [A]

16 If the graph turns out to be straight-line and passes through the origin, then the suggested relationship is correct. [A]

Safety Precautions:

- To avoid splashing, I will drop the ball near the oil surface. [S]
- To prevent falling and rolling of the steel balls on the floor, I will keep them in a tray. [D]

Further additional-detail points might include:

- Allow oil to stand so that air bubbles escape/ball may trap air bubbles.

Note: A candidate can score only **four** D-marks at the most. However, as there is no negative marking in this paper, candidates are advised to provide as much additional detail in their accounts as they can think of.

Sample Question 1.2 *(P52/M/J/11)*

A student wishes to investigate projectile motion. A small ball is rolled with velocity v along a horizontal surface. When the ball reaches the end of the horizontal surface, it falls and lands on a lower horizontal surface. The vertical displacement of the ball is p and the horizontal displacement of the ball is q, as shown in Fig. 1.2.1.

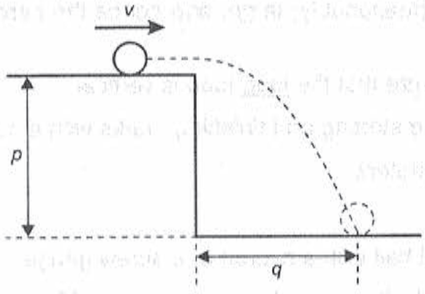


Figure 1.2.1

It is suggested that:

$$gq^2 = 2pv^2$$

where g is the acceleration of free fall. Design a laboratory experiment to investigate how q is related to p and how v may be determined from the results. You should draw a diagram showing the arrangement of your equipment. In your account you should pay particular attention to:

(a) the procedure to be followed,

(b) the measurements to be taken,

(c) the control of variables,

(d) the analysis of the data,

(e) the safety precautions to be taken. [15]

Solution

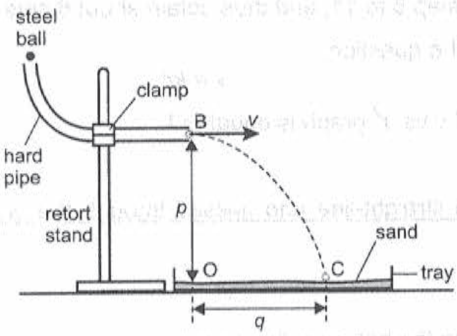


Figure 1.2.2

In this experiment, I will:

- vary p and measure q (for each value of p). [P + P]

  (i.e. p is the independent variable, and q is the dependent variable.)

- keep (horizontal) velocity v constant. [P]

To collect and analyse the data, I will take the following steps:

1 Build the experimental setup as shown in Fig. 1.2.2. In the experimental setup:

- the purpose of using the arrangement of retort stand, clamp and hard pipe is to vary p. [M]
- the purpose of using the tray of sand is to determine the position of the ball as it lands on the sand surface. [M]
- the purpose of using the hard pipe, which is half curved and half straight, is to make sure that the velocity of the ball, as it leaves the pipe, has horizontal component only. [D]

When building the experimental setup, I will:

- use a spirit level to make sure that the straight part of the pipe is horizontal.
- use a plumb line to draw, on the sand surface, a small mark O right below the end B of the pipe. [D]

2 Measure distance p between end B and mark O with a metre rule. [M]

3 Take a steel ball to minimise the effect of air resistance. [D]

4 Release the ball from the top end A of the pipe. The ball will roll, fall and land on the sand surface producing a crater at point C.

5 Measure the distance q between point C and mark O with a metre rule. [D]

6 Repeat steps **4** and **5** for same p and find the average value of q. [D]

7 Change the height of the clamp so that the end B remains right above the mark O.

8 Repeat the procedure from step **4** to **6**, and thus obtain about 6 sets of results. When repeating the procedure to collect the data, I will:

- always use the spirit level to make sure that the straight part of the pipe is horizontal. [M]
- always release the ball from the top end A of the pipe (to make sure that the velocity v remains constant). [M]

9 The equation given in the question can be rearranged as:

$$q^2 = \left(\frac{2v^2}{g}\right)p$$

From the above equation, it follows that the gradient of q^2 vs. p graph is equal to the expression: $\frac{2v^2}{g}$.

10 Plot a graph of q^2 against p. [A]

11 If the graph turns out to be straight-line and passes through the origin, then the suggested relationship is correct. [D]

12 Find the gradient of the graph, and determine v using the equation:

$$v = \sqrt{\frac{g \times \text{gradient}}{2}}$$ [A]

Safety Precaution:

To prevent injury from the rolling ball, I will use safety screen. [S]

Further additional-detail points might include:

- Detail on method of determining position of ball; e.g. slow motion playback including scale.

Sample Question 1.3 *(P51/M/J/10)*

A hammer is often used to force a nail into wood. The faster the hammer moves, the deeper the nail moves into the wood. This can be represented in a laboratory by a mass falling vertically onto a nail. It is suggested that the depth d of the nail in the wood (see Fig. 1.3.1) is related to the velocity v of the mass at the instant it hits the nail by the equation:

$$d = kv^n$$

where k and n are constants.

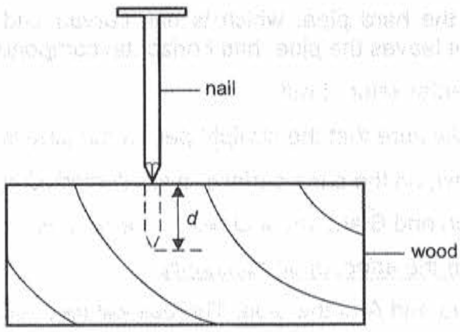


Figure 1.3.1

Design a laboratory experiment to investigate the relationship between v and d so as to determine a value for n. You should draw a diagram showing the arrangement of your equipment. In your account you should pay particular attention to:

(a) the procedure to be followed,

(b) the measurements to be taken,

(c) the control of variables,

(d) the analysis of the data,

(e) the safety precautions to be taken. [15]

Solution

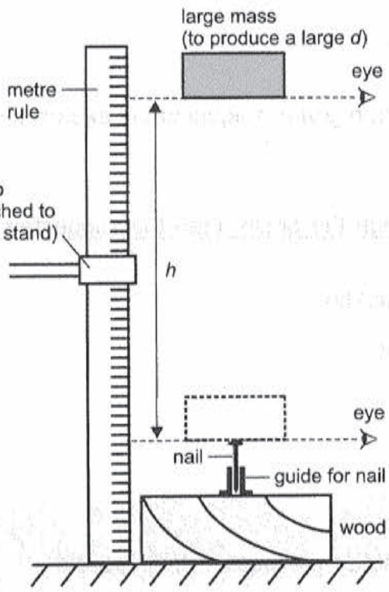


Figure 1.3.2

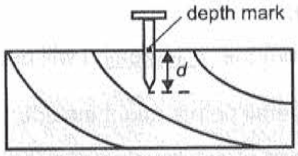


Figure 1.3.3 (enlarged view)

In this experiment, I will:

- vary v and measure d (for each value of v). [P]
  (i.e. v is the independent variable, and d is the dependent variable.)
- keep the mass (falling onto the nail) constant. [P]
- also keep the wood constant. [P]
  (i.e. use the same wood type.)

To collect and analyse the data, I will take the following steps:

1 Build the experimental setup as shown in Fig. 1.3.2. In the experimental setup:
  - the mass is positioned above the nail so that it falls onto the centre of the nail when dropped. [M]
  - the purpose of using the vertical guide, the internal diameter of which is only slightly greater than the diameter of the nail, is to make sure that the nail goes straight into the wood when hit by the falling mass. [D]
  - the purpose of using the large mass is to produce a large depth d, and thus reduce the percentage uncertainty in its measurement. [D + D]

  When building the experimental setup, I will use a set square to make sure that the nail is vertical. [D]

2 Hold the mass at a height h above the head of the nail.

3 Measure h with a metre rule. [M]
  When measuring h, I will avoid parallax error by keeping the eye at right level as shown in Fig. 1.3.2. [D]

4 Drop the mass from height h. The mass will fall, hit and force the nail into the wood to depth d.

5 Determine the velocity v of the mass with which it hits the nail using the equation:

$$v^2 - u^2 = 2gh$$

  where $u = 0$ and $g = 9.81\ \text{m s}^{-2}$. [M]

6 Mark the nail (with a thin hacksaw blade) at a point up to which it goes into the wood, as shown in Fig. 1.3.3, (and then pull it out). [M]

7 Measure the depth d with vernier calipers. [D]

8 Repeat steps **4, 6,** and **7** for same h and find the average value of d. [D]
  When repeating the steps **4, 6** and **7**, I will use different part of the wood every time. [D]

9 Change the height of the falling mass (to vary v). [M]

10 Repeat the procedure from step **3** to **8**, and thus obtain about 6 sets of results.

11 From the equation given in the question, it can be shown that:

$$\lg d = n \lg v + \lg k$$

  [D]
  From the above equation, it follows that the gradient and y-intercept of $\lg d$ vs. $\lg v$ graph are equal to n and $\lg k$ respectively.

12 Plot a graph of $\lg d$ against $\lg v$. [A]

13 If the graph turns out to be straight-line, then the suggested relationship is correct. [D]
  (**Note:** The graph line might or might not pass through the origin; it all depends upon the value of k. For example: if $k = 1$, then the graph will pass through the origin, as $\lg 1 = 0$; otherwise not.)

14 Determine the value of n by finding the gradient of the graph, as:

$$n = \text{gradient}$$

  [A]

Safety Precaution:

To prevent injury from the falling mass, I will keep my hands and feet well away from it. [S]

Further additional-detail points might include:

- Use of microscope when measuring d.

Sample Question 1.4 *(P51/M/J/13)*

A student is investigating the flow of water through a horizontal tube. The rate Q (volume per unit time) at which water flows through a tube depends on the pressure difference per unit length across the tube. The student has the use of a metal can with two holes. A narrow horizontal tube goes through the hole in the side of the can. The can is continuously supplied with water from a tap. The level of water in the can is kept constant by the position of a wide vertical tube which passes through the hole in the bottom of the can as shown in Fig. 1.4.1. Both tubes may be moved along the holes.

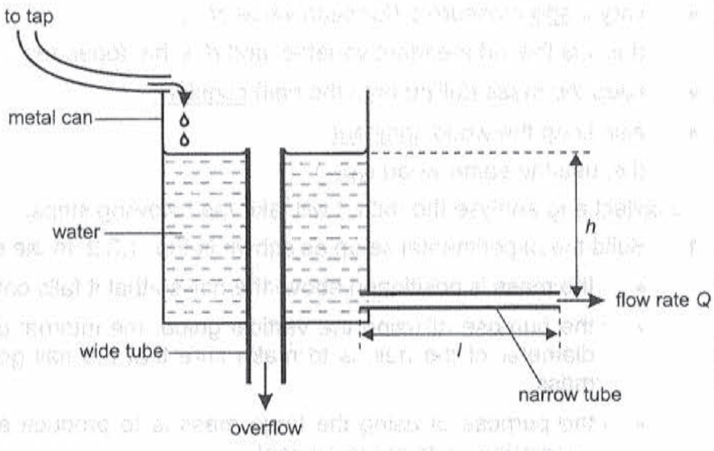


Figure 1.4.1

It is suggested that the relationship between the flow rate Q of water through the narrow horizontal tube and the vertical height h is:

$$Q = \frac{2\pi\rho g h d^4}{\eta l}$$

where ρ is the density of water, g is the acceleration of free fall, d is the internal diameter of the tube, l is the length of the tube and η is a constant. Design a laboratory experiment to test the relationship between Q and h and determine a value for η. You should draw a diagram showing the arrangement of your equipment. In your account you should pay particular attention to:

(a) the procedure to be followed,

(b) the measurements to be taken,

(c) the control of variables,

(d) the analysis of the data,

(e) the safety precautions to be taken. [15]

Solution

In this experiment, I will:

- vary h and determine Q (for each value of h). [P + P]
- keep l <u>constant</u>. [P]
- also keep the <u>temperature of water constant</u>. [D]
- (so that the density ρ of water remains constant.)

To collect and analyse the data, I will take the following steps:

1 Take some water in a measuring cylinder and record its volume V_w.

2 Measure the combined mass m_{cw} of the cylinder and water with a digital balance.

3 Empty the cylinder and measure its mass m_c with the digital balance.

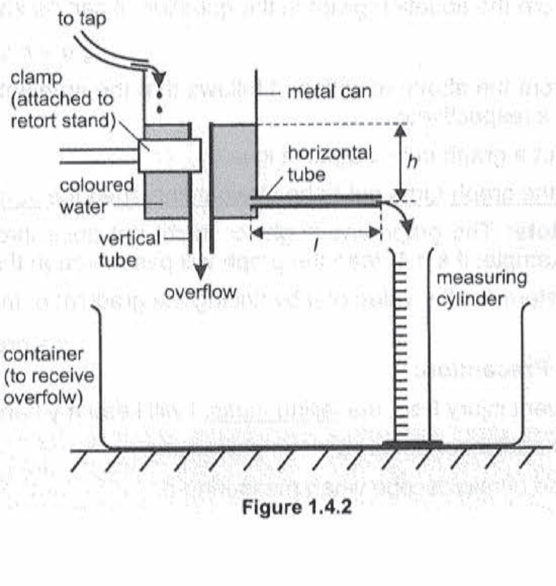


Figure 1.4.2

4 Find the mass m_w of the water using the equation:

$$m_w = m_{cw} - m_c$$

5 Determine the density ρ of water using the formula:

$$\rho = \frac{m_w}{V_w}$$ [D]

(**Note:** All steps, from **1** to **5**, are required to score this mark.)

6 Measure the internal diameter d of the narrow tube with vernier calipers. [M]

When measuring d, I will take multiple readings in different directions and find the average value. [D]

7 Measure the length l of the narrow tube with a rule.

8 Build the experimental setup as shown in Fig. 1.4.2. In the experimental setup:

- the purpose of using the measuring cylinder is to receive and measure the volume of water flowing out of the narrow tube. [M]
- the purpose of using the coloured water is to take the reading of volume of water with ease. [D]
- the purpose of using the large container is to receive the overflow. [D]
- the purpose of using the retort stand and clamp arrangement is to hold the steel can in position above the container. [D]

When building the experimental setup, I will use a spirit level to make sure that the narrow tube is horizontal. [D]

9 Measure the vertical height h of the wide tube with the rule. [M]

10 Note the capacity V of the measuring cylinder.

11 With a stopwatch, measure the time t taken for the water flowing out of the horizontal tube to fill the empty measuring cylinder. [M]

12 Repeat step **11** for same h and find the average value of t. [D]

13 Determine the flow rate Q of water through the horizontal tube using the formula:

$$Q = \frac{V}{t}$$

14 Change the position of the vertical tube (to vary h). [M]

15 Repeat the procedure from step **9** to **13**, and thus obtain about 6 sets of results.

16 From the equation given in the question:

$$Q = \left(\frac{2\pi\rho g d^4}{\eta l}\right)h$$

it follows that the gradient of Q vs. h graph is equal to the expression: $\frac{2\pi\rho g d^4}{\eta l}$.

17 Plot a graph of Q against h. [A]

18 If the graph turns out to be straight-line and passes through the origin, then the suggested relationship is correct. [D]

19 Find the gradient of the graph, and determine the value of η using the equation:

$$\eta = \frac{2\pi\rho g d^4}{l \times \text{gradient}}$$ [A]

Safety Precaution:

To prevent injury when adjusting the metal tubes, I will wear (protective) gloves. [S]

Further additional-detail points might include:

- Use a large measuring cylinder to reduce percentage uncertainty in the measurement of time t / flow rate Q.
- Detail on measuring h to the centre of the horizontal tube; e.g. add radius to tube.

Sample Question 1.5 *(P05/M/J/09)*

A student wishes to determine the Young modulus E of wood from the period of oscillation of a loaded wooden rule, as shown in Fig. 1.5.1.

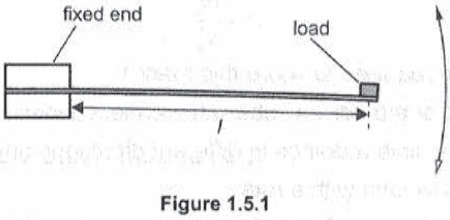


Figure 1.5.1

An equation relating the period of oscillation T to the overhanging length l of the rule is:

$$T^2 = \frac{kl^3}{E}$$

The constant k is given by:

$$k = \frac{16\pi^2 M}{wd^3}$$

where M is the mass of the load, w is the width of the rule and d is the thickness of the rule. Design a laboratory experiment to determine the Young modulus of wood. You should draw a diagram showing the arrangement of your equipment. In your account, you should pay particular attention to:

(a) the procedure to be followed,

(b) the measurements to be taken,

(c) the control of variables,

(d) how to analyse the data,

(e) how to determine E,

(f) the safety precautions to be taken. [15]

Solution

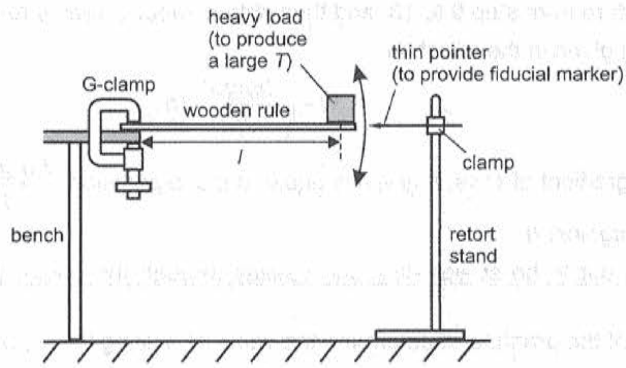


Figure 1.5.2

In this experiment, I will:

- vary l and determine T (for each value of l). [P + P]
- keep M constant. [P]
- also keep w and d constant. [D]

  (i.e. use the rule having uniform w and d along the length.)

To collect and analyse the data, I will take the following steps:

1 Take a load of a large mass M to produce a measurable period of oscillation T. [D]

2 Measure the mass M of the load with a digital balance. [M]

3 Take a half-metre wooden rule.

4 Measure the width w and thickness d of the rule with vernier calipers. [M]

When measuring w and d, I will take multiple readings at different points along the rule and find the average values. [D]

5 Build the experimental setup as shown in Fig. 1.5.2. In the experimental setup:

- the purpose of using the bench and G-clamp arrangement is to fix one end of the rule firmly in position. [M]
- the purpose of using the thin pointer, positioned close to the equilibrium position of the load, is to provide fiducial marker; so that the oscillations may be timed with ease. [D]

When building the experimental setup, I will secure the load to the rule with tape. [D]

6 Record the overhanging length l of the rule. [M]

7 Set the rule into oscillation, while keeping the amplitude of oscillation (reasonably) small (to ensure that the equation relating T and l holds good.) [D]

8 Wait until the oscillations have settled. [D]

9 With a stopwatch, time at least 10 oscillations, so that the time t taken for 10 oscillations is (reasonably) large and hence the percentage uncertainty in its measurement is (reasonably) small. [D]

10 Repeat step **9** and find the average value of t. [M]

11 Determine the period of oscillation T using the formula:

$$T = \frac{t}{10}$$

12 Change the position of the load on the rule (to vary l).

13 Repeat the procedure from step **6** to **11**, and thus obtain about 6 sets of results.

14 From the two equations given in the question, it can be shown that:

$$T^2 = \left(\frac{16\pi^2 M}{wd^3 E}\right) l^3$$

From the above equation it follows that the gradient of T^2 vs. l^3 graph is equal to the expression: $\frac{16\pi^2 M}{wd^3 E}$.

15 Plot a graph of T^2 against l^3. [A]

16 If no mistake is made up to this point, then the graph will be straight-line and pass through the origin. [D]

17 Find the gradient of the graph, and determine the Young modulus E of wood using the equation:

$$E = \frac{16\pi^2 M}{wd^3 \times \text{gradient}}$$ [A]

Safety Precaution:

To prevent injury from the load, which may detach from the rule during oscillation, I will keep my feet well away from it. [S]

Further additional-detail points might include:

- Discussion of use of motion sensor, e.g. orientation, or light gates with detail.

Sample Question 1.6 *(P53/O/N/10)*

A student wishes to determine the resistivity of aluminium. The resistivity ρ of a conductor is defined as:

$$\rho = \frac{RA}{l}$$

for a conductor of resistance R, cross-sectional area A and length l. Fig. 1.6.1 shows the typical dimensions of a strip of aluminium of lengths c, d and t. The resistivity of aluminium is about 10^{-8} Ω m.

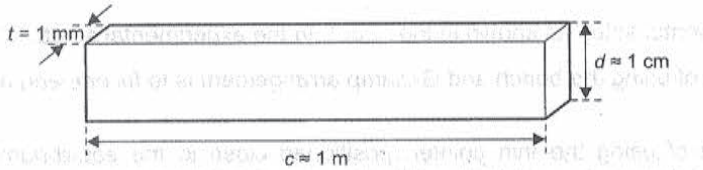


Figure 1.6.1 (not to scale)

Design a laboratory experiment to determine the resistivity of aluminium using this strip. The usual apparatus of a school laboratory is available, including a metal cutter. You should draw a diagram showing the arrangement of your equipment. In your account you should pay particular attention to:

(a) the procedure to be followed,

(b) the measurements to be taken,

(c) the control of variables,

(d) the analysis of the data,

(e) the safety precautions to be taken. [15]

Solution

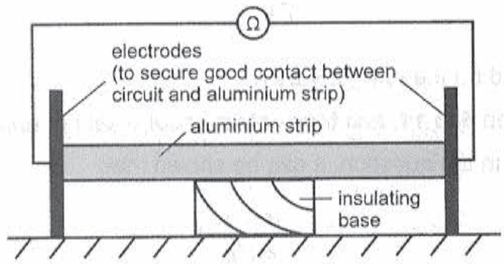


Figure 1.6.2

In this experiment, I will:

- vary c and determine R (for each value of c). [P]
- keep (t) and d constant. [P]
- also keep temperature of the strip constant. [P]

To collect and analyse the data, I will take the following steps:

1 Measure the thickness t and width d of the strip with a micrometer screw gauge. [M]

When measuring t and d, I will take multiple readings at different points along the strip and find the average values. [D]

2 Determine the average cross-sectional area A of the strip by multiplying t and d together.

3 Measure the length c of the strip with a metre rule. [M]

4 Build the experimental setup as shown in Fig. 1.6.2. In the experimental setup:

- the purpose of using the ohmmeter is to measure the resistance of the strip. [M]

  (**Note:** It is not necessary to label those components of the circuit that have already been represented by their conventional symbols.)

- the purpose of using the electrodes (metal plates/rods) is to secure good contact between the circuit and the strip. [M]
- the purpose of using the wooden block is to insulate the strip). [D]

When setting up the circuit, I will also use conducting putty to make sure that the whole cross-sectional area of the ends of the strip is in good contact with the electrodes. [D]

5 Record the resistance R of the strip from the ohmmeter. [M]

6 Using a set square draw, on the strip, a straight mark (about 15 cm from either end) perpendicular to its length, and then cut the strip along the mark with a metal cutter. [D]

7 Repeat the procedure from step **3** to **5**, and thus obtain about 6 sets of results.

8 The equation given in the question can be rearranged as:

$$R = \left(\frac{\rho}{A}\right)c$$

From the above equation, it follows that the gradient of R vs. c graph is equal to the expression: $\frac{\rho}{A}$.

9 Plot a graph of R against c. [A]

10 If no mistake is made up to this point, then the graph will be straight-line and pass through the origin. [D]

11 Find the gradient of the graph, and determine the resistivity ρ of aluminium using the equation:

$$\rho = \text{gradient} \times A$$ [A]

Safety Precaution:

To prevent cuts from the sharp edges of the strip, I will wear (protective) gloves. [S]

Further additional-detail points might include:

- Determination of a typical resistance of aluminium strip using values of dimensions given.
- Likely meter range of ohmmeter/ammeter/voltmeter with reasoning.

  Note: Usually, a measuring instrument with smaller range is more sensitive (i.e. has smaller least count). An appropriate measuring instrument is the one the (maximum) range of which is not much greater than the maximum possible value of the physical quantity to be measured. Such instrument gives least percentage uncertainty in the measurement of the physical quantity measured.

- Use a protective resistor to reduce current/heating effect (if power supply, ammeter and voltmeter circuit is used to determine the resistance).

Sample Question 1.7 *(P05/M/J/08)*

A student wishes to measure the resistivity of glass. A teacher suggests that its resistivity is of the order of 10^6 Ω m which is very large. Resistivity ρ is defined by the equation:

$$\rho = \frac{RA}{l}$$

where R is resistance, A is cross-sectional area and l is the length of the material. The student is given a number of sheets of glass of the same thickness and of different areas. Design a laboratory experiment to determine the resistivity of glass. You should draw a diagram showing the arrangement of your equipment. In your account you should pay particular attention to:

(a) the procedure to be followed,

(b) how the glass would be connected to the circuit,

(c) the measurements that would be taken,

(d) the control of variables,

(e) how the data would be analysed,

(f) any safety precautions that you would take. [15]

Solution

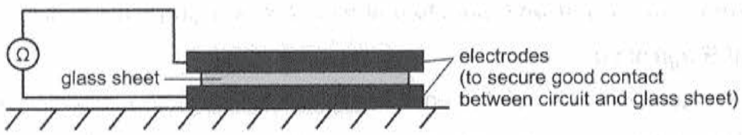


Figure 1.7.1

In this experiment, I will:

- vary A and determine R (for each value of A). [P + P]
- keep temperature of the glass sheets constant. [P]

To collect and analyse the data, I will take the following steps:

1. Wash and dry the glass sheets. [D]
2. Measure the thickness l of a glass sheet with a micrometer screw gauge. [M]

   When measuring l, I will take multiple readings (using different glass sheets) and find the average value. [D]
3. Take a glass sheet and determine its area A, which is to be kept perpendicular to the direction of flow of current, by first measuring the respective side lengths with a metre rule and then multiplying them together. [M]
4. Build the experimental setup as shown in Fig. 1.7.1. In the experimental setup:
   - the purpose of using the ohmmeter is to measure the resistance of the sheet. [M]
   - the glass sheet is oriented so that it offers minimum resistance. [M]
   - the purpose of using the electrodes (metal plates) is to secure good contact between the circuit and glass sheet. [D]

   When setting up the circuit, I will also use conducting putty to make sure that the whole area A of the sheet is in good contact with the electrodes. [D]
5. Record the resistance R of the sheet from the ohmmeter. [M]
6. Take another glass sheet of different area A.

7 Repeat the procedure from step **3** to **5**, and thus obtain about 6 sets of results.

8 The equation given in the question can be rearranged as:

$$R = \rho l\left(\frac{1}{A}\right)$$

From the above equation, it follows that the gradient of R vs. $(1/A)$ graph is equal to the expression 'ρl'.

9 Plot a graph of R against $(1/A)$. [A]

10 If no mistake is made up to this point, then the graph will be straight-line and pass through the origin. [D]

11 Find the gradient of the graph, and determine the resistivity ρ of glass using the equation:

$$\rho = \frac{\text{(gradient)}}{l}$$ [A]

Safety Precautions:

- To prevent cuts when handling the glass sheets, I will wear (thick) gloves. [S]

Further additional-detail points might include:

- Determination of a typical resistance of glass using value of resistivity given.
- Use of EHT (an extremely high voltage source) or power supply > 100 V or micro-ammeter/galvanometer (as the resistance of the glass sheet is very high, so very high voltage is required to produce measurable current in the circuit).
- Likely meter range of ammeter or ohmmeter with reasoning.
- Additional method of securing good contact between circuit and glass sheet, e.g. use of G-clamp or weight.

Note: If EHT power supply, ammeter and voltmeter circuit is used to determine the resistance of the glass sheet, then the safety precaution should be related to the EHT power supply; such as

- To prevent electric shock when changing the circuit, I will switch off the EHT power supply first/wear rubber gloves.

Sample Question 1.8 *(P53/O/N/13)*

A student is investigating how the resistance R of nichrome in the form of a wire varies with temperature θ. It is suggested that:

$$R = R_0(1 + \alpha\theta)$$

where R_0 is the resistance at 0 °C, α is a constant and θ is the temperature measured in °C. Design a laboratory experiment to test the relationship between θ and R and determine the value of α. You should draw a diagram showing the arrangement of your equipment. In your account you should pay particular attention to:

(a) the procedure to be followed,

(b) the measurements to be taken,

(c) the control of variables,

(d) the analysis of the data,

(e) the safety precautions to be taken. [15]

Solution

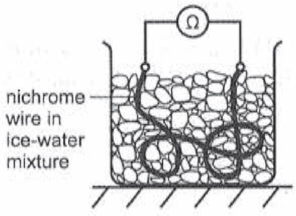


Figure 1.8.1

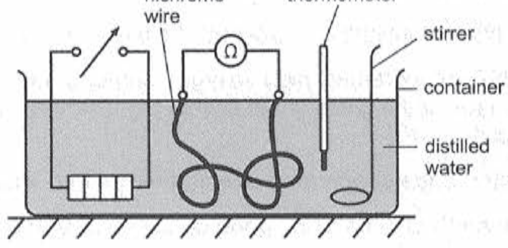


Figure 1.8.2

In this experiment, I will:

- vary θ and determine R (for each value of θ). [P + P]
- keep length of the wire constant. [P]

To collect and analyse the data, I will take the following steps:

1. Take a long nichrome wire to obtain a large value for its resistance and hence a small percentage uncertainty in the measurement of resistance. [D + D]
2. Immerse it in the ice-water mixture as shown in Fig. 1.8.1. [M]

   (The temperature of melting ice is always 0 °C)
3. Wait for its temperature to become 0 °C. [D]
4. Record its resistance R_0 from the ohmmeter. [M]
5. Take the wire out of the ice-water mixture and build the experimental setup as shown in Fig. 1.8.2. In the experimental setup:
   - the purpose of using the heater is to supply heat to the nichrome wire through water. [M]
   - the purpose of using the stirrer is to make sure that the heat is distributed uniformly. [D]
   - the purpose of using the thermometer is to measure the temperature of the wire. [M]
   - the purpose of using the ohmmeter is to measure the resistance of the wire. [M]
6. Switch on the variable power supply connected to the heater. The temperature of the water (and everything immersed in it) will start rising.

7 Wait for the reading of the thermometer (i.e. temperature of the water and wire) to stabilise.

8 Record the temperature θ of the wire from the thermometer.

9 Record the resistance R of the nichrome wire from the ohmmeter.

10 Turn up the heater a bit. The temperature of the water will start rising again.

11 Repeat the procedure from step **7** to **9**, and thus obtain about 6 sets of results.

12 The equation given in the question can be rearranged as:

$$R = (R_0\alpha)\theta + R_0$$

From the above equation, it follows that the gradient and *y*-intercept of R vs. θ graph are equal to '$R_0\alpha$', and R_0 respectively.

13 Plot a graph of R against θ. [A]

14 If the graph turns out to be straight-line and does not pass through the origin, then the suggested relationship is correct. [D + D]

15 Find the gradient of the graph, and determine the value of α using the equation:

$$\alpha = \frac{(\text{gradient})}{R_0}$$ [A]

Safety Precaution:

To prevent burns from hot nichrome wire, I will wear gloves. [S]

Further additional-detail points might include:

- Use insulated wire.
- Use a protective resistor to minimise heating effect (if power supply, ammeter and voltmeter circuit is used to determine the resistance).

Sample Question 1.9 *(P52/O/N/09)*

When a current passes through a wire, the wire becomes hot and expands. This can be investigated in a laboratory by passing a current through a wire of diameter d and measuring the displacement y, as shown in Fig. 1.9.1.

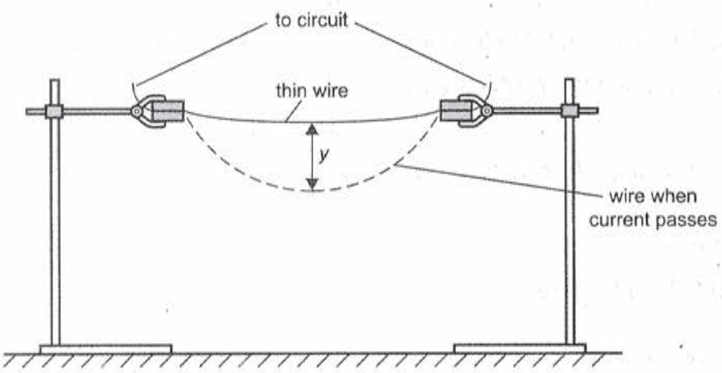


Figure 1.9.1

It is suggested that the diameter d of the wire is related to y by the equation:

$$y = pd^q$$

where p and q are constants. Design a laboratory experiment to investigate the relationship between d and y, so as to determine a value for q. You should draw a diagram showing the arrangement of your equipment. In your account you should pay particular attention to:

(a) the procedure to be followed,

(b) the measurements to be taken,

(c) the control of variables,

(d) the analysis of the data,

(e) the safety precautions to be taken. [15]

Solution

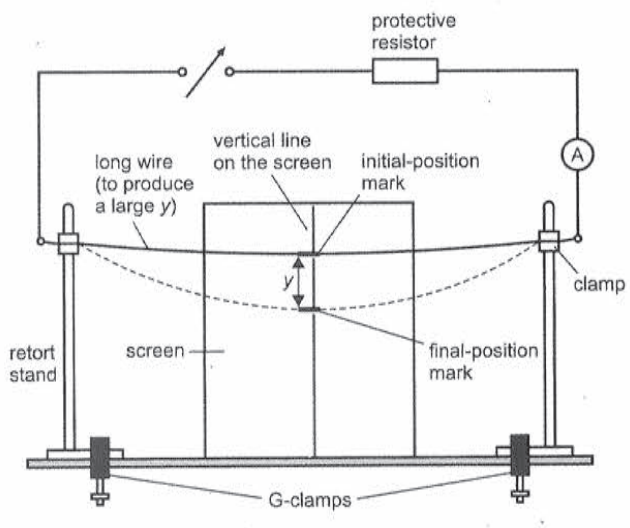


Figure 1.9.2

In this experiment, I will:

- vary d and measure y (for each value of d). [P]
- keep (original) length of the wire (between the clamps), and current (through it) constant. [P + P]
- also keep room temperature constant. [D]

To collect and analyse the data, I will take the following steps:

1 Measure the diameter d of the wire with a micrometer screw gauge. [M]

When measuring d, I will take multiple readings at different points along the wire and find the average value. [D]

2 Build the experimental setup as shown in Fig. 1.9.2. In the experimental setup:

- the purpose of using the variable power supply is to supply current to the wire, and the purpose of using the screen is to mark the initial and final positions of the wire. [M]
- the purpose of using the ammeter is to check the current flowing through the wire. [M]
- the purpose of using the protective resistor is to reduce the current to minimise the heating effect. [D]
- the purpose of using the G-clamps is to keep the distance between the retort stands constant so that the length of the wire between the clamps remains constant. [D]
- the purpose of using the long wire is to produce a large y and thus reduce the percentage uncertainty in the measurement of y. [D + D]

When building the experimental setup, I will draw a vertical line on the white screen with the help of a set square, and position the screen so that the vertical line is midway between the clamps. [D]

3 Draw, on the vertical line, a small mark to indicate the initial position of the wire, while keeping the eye at the level of the wire to avoid parallax error. [D]

4 Switch on the variable power supply. A current will start flowing through the wire, and it will start undergoing expansion.

5 Wait for the wire to stabilise, and then mark its final position on the screen the same way as in step **3**. [M]

6 Determine the displacement y by measuring the distance between the initial and final position marks with vernier calipers. [M + D]

7 Take another wire of different diameter.

8 Repeat the procedure from step **1** to **6**, and thus obtain about 6 sets of results.

9 When repeating the procedure to collect the data, I will:

- always adjust the output voltage of the variable power supply to keep the current through the wire constant. [D]
- always check the starting position, for y, of each wire. [D]

10 From the equation given in the question, it can be shown that:

$$\lg y = q \lg d + \lg p$$ [D]

From the above equation, it follows that the gradient and y-intercept of $\lg y$ vs. $\lg d$ graph are equal to q and $\lg p$ respectively.

11 Plot a graph of $\lg y$ against $\lg d$. [A]

12 If the graph turns out to be straight-line, then the suggested relationship is correct. [D]

13 Determine the value of q by finding the gradient of the graph, as:

$$q = \text{gradient}$$ [A]

Safety Precaution:

To prevent burns from the hot wire, I will wear gloves. [S]

Sample Question 1.10 *(P05/O/N/08)*

A student wishes to investigate how the resistance R of a light-dependent resistor varies with the distance d from an intense light source. It is believed that the relationship between R and d is:

$$R = kd^n$$

where k and n are constants. Design a laboratory experiment to test the above relationship. The light-dependent resistor has a resistance of 100 Ω when it is in bright light and a resistance of 500 kΩ when no light falls on it. You should draw a diagram showing the arrangement of your equipment. In your account you should pay particular attention to:

(a) the procedure to be followed,

(b) the measurements that would be taken,

(c) the control of variables,

(d) how the data would be analysed,

(e) any safety precautions that you would take. [15]

Solution

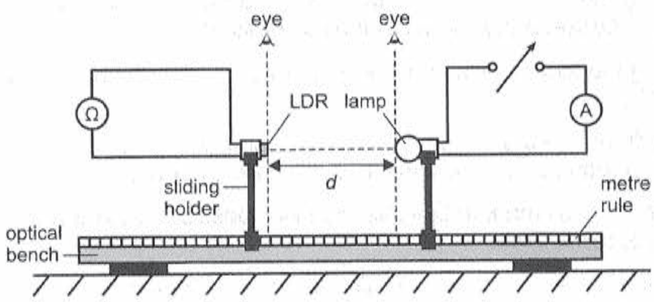


Figure 1.10.1

In this experiment, I will:

- vary d and measure R (for each value of d). [P + P]
- keep current through the light source (lamp) constant; [P]
  (so that its brightness, or the intensity of light emitted, remains constant)
- also keep temperature of the light-dependent resistor (LDR) constant. [D]
- also keep the orientation of the LDR, with respect to the lamp, constant. [D]

To collect and analyse the data, I will take the following steps:

1 Choose a dark room to perform the experiment so that there is no source of light other than that used on purpose (i.e. lamp), as the resistance of LDR changes with intensity of light falling on it. [M + D]

2 Build the experimental setup as shown in Fig. 1.10.1. In the experimental setup:
- the purpose of using the independent lamp is to shine light on the LDR connected into a separate circuit. [M]
- the purpose of using the metre rule fixed to the optical bench is to measure the distance d between the lamp and LDR. [M]
- the purpose of using ohmmeter is to measure the resistance R of the LDR. [M]
- the purpose of using the ammeter is to check if the current through the lamp remains constant. [D]
- the purpose of using the variable power supply is to keep the current through the lamp constant. [D]
- the purpose of using the optical bench and sliding holders is to keep the orientation of the LDR, with respect to the lamp, constant. [D]

3 Note the readings, on the metre rule, of the positions of the lamp and the LDR by looking from above, and determine *d* by calculating the difference between the readings. [D]

When measuring *d*, I will avoid parallax error by keeping the eye in the right position as shown in Fig. 1.10.1. [D]

4 Record the resistance *R* of the LDR from the ohmmeter. [M]

5 Change *d* by sliding the holder holding LDR.

6 Repeat the procedure from step **3** to **4**, and thus obtain about 6 sets of results.

7 From the equation given in the question, it can be shown that:

$$\lg R = n \lg d + \lg k$$ [D]

From the above equation, it follows that the gradient and *y*-intercept of lg *R* vs. lg *d* graph are equal to '*n*' and 'lg *k*' respectively.

8 Plot a graph of lg *R* against lg *d*. [A]

9 If the graph turns out to be straight-line, then the given relationship is correct. [A]

Safety Precaution:

To prevent burns from the hot lamp, I will wear gloves. [S]

Further additional-detail points might include:

- Determination of a typical current using value of resistance given.
- Likely meter range of ohmmeter or ammeter with reasoning.

Note: From this point onwards, only complete labelled diagrams of the experimental arrangements and marking schemes (with explanation where required) have been given in the solution sections of the sample questions.

Sample Question 1.11 *(P51/M/J/11)*

When light is incident on the front of a photocell, an e.m.f. is generated in the photocell. A student wishes to investigate the effect of adding various thicknesses of glass in front of a photocell. This may be carried out in the laboratory by varying the number of identical thin glass sheets between a light source and the front of the photocell. It is suggested that the e.m.f. V is related to the number n of glass sheets by the equation:

$$V = V_0 e^{-\alpha n t}$$

where t is the thickness of one sheet, α is the absorption coefficient of glass and V_0 is the e.m.f. for $n = 0$. Design a laboratory experiment to determine the absorption coefficient of glass. You should draw a diagram showing the arrangement of your equipment. In your account you should pay particular attention to:

(a) the procedure to be followed,

(b) the measurements to be taken,

(c) the control of variables,

(d) the analysis of the data,

(e) the safety precautions to be taken. [15]

Solution

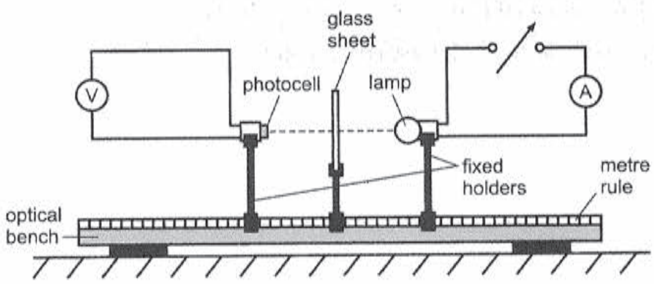


Figure 1.11.1

Defining the problem (3 P-marks)

- n is the independent variable and V is the dependent variable, or vary n and measure V (for each value of n).
- Keep brightness/intensity of light/power output of lamp/current through lamp/voltage across lamp constant.
  Note: no mark for stating: 'same lamp'.
- Keep distance from light to photocell constant.
  Note: no mark for stating: 'control' instead of 'constant'.

Methods of data collection (5 M-marks)

- Labelled diagram of apparatus: lamp, glass sheet and photocell in line.
- Voltmeter connected to photocell.
- Use micrometer (screw gauge) to measure thickness of glass sheet.
- Take many readings of thickness (using different glass sheets) and average.
- Perform experiment in a dark room, or shield apparatus.

Method of analysis (2 A-marks)

	or
• Plot a graph of ln V against n	ln V against nt
• α = (–) gradient / t	α = (–) gradient

Note: Any combination of functions of dependent and independent variables, the graph of which is possible to plot and would produce a straight line is acceptable.

Safety considerations (1 S-mark)

- To prevent burns from hot source, wear gloves/switch off when not in use/avoid touching.
- To prevent eye damage from bright/intense source, wear dark glasses/do not look at source directly/shield lamp.
- To prevent cuts from glass, wear gloves.

Additional detail (4 D-marks)

- Clean glass sheets before use.
- Measurement of V_0 stating that no glass sheet is present.
- Keep orientation of photocell and glass sheets, with respect to light source, constant.
- Method for keeping orientation of photocell and glass sheets, with respect to light source constant, e.g. use optical bench, or use set square, or fix to rule.
- Use small distance/high intensity to produce a large V.
- Produce a large V to reduce percentage uncertainty in its measurement.
- Method to check brightness of lamp is constant, e.g. measure current through lamp regularly, or measure p.d. across lamp regularly, or check V_0 with no glass regularly.
- Method to ensure brightness of lamp is constant, e.g. workable circuit diagram with variable power supply or variable resistor.
- Method to check distance of photocell from light source is constant, e.g. use an arrangement of optical bench, metre rule and fixed holders.
- Reason for performing experiment in a dark room related to the photocell.
- $\ln V = -\alpha nt + \ln V_0$.
- Relationship is valid if graph is a straight line with y-intercept = ln V_0 (provided plotted graph is correct).
- Further safety consideration (from the ones stated under safety considerations' section).

Sample Question 1.12 *(P53/O/N/12)*

Two identical light sources are viewed from a distance, as shown in Fig. 1.12.1. When the angle θ between the light sources is large, they are seen as separate.

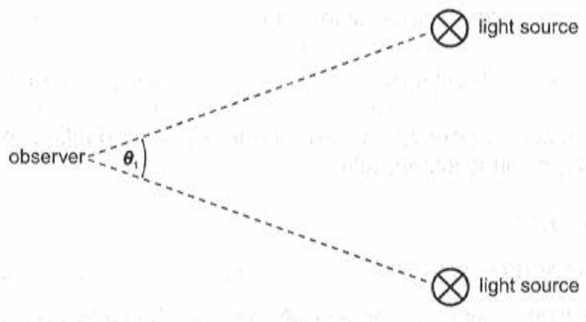


Figure 1.12.1

The sources are moved closer together. At a particular angle θ_1 the two sources appear as a single source. It is suggested that θ_1 is directly proportional to the wavelength λ of the light from the sources. Design a laboratory experiment using two light sources to test the relationship between θ_1 and λ. You should draw a diagram showing the arrangement of your equipment. In your account you should pay particular attention to:

(a) the procedure to be followed,

(b) the measurements to be taken,

(c) the control of variables,

(d) the analysis of the data,

(e) the safety precautions to be taken. [15]

Solution

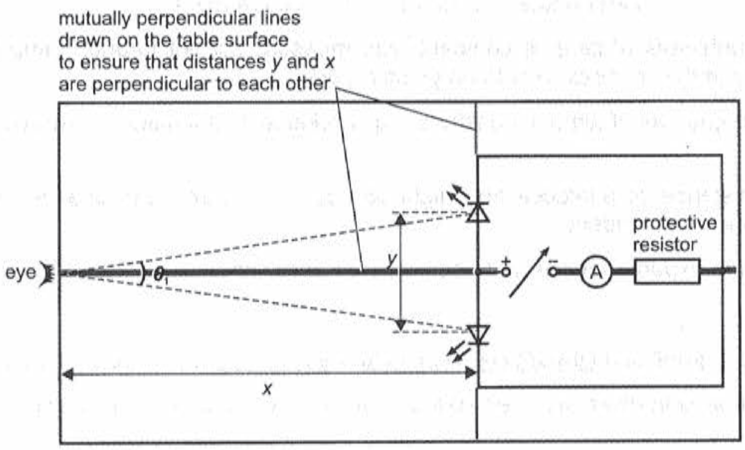


Figure 1.12.2 (top view of table surface)

Defining the problem (3 P-marks)

- λ is the independent variable, or vary λ (i.e. use several pairs of light sources of different wavelengths).
- θ_1 is the dependent variable, or measure θ_1 (for each value of λ).
- Keep brightness/intensity of light sources/power output of light sources/current through them/voltage across them constant.

Methods of data collection (5 M-marks)

- Labelled diagram showing observer (i.e. eye), light sources with method of producing monochromatic light, e.g. coloured LED/filter.
- Method to measure wavelength, e.g. record from LED/filter or Young's slit/diffraction grating method.
- Use a rule/measuring tape to measure distances (x and y).
- Method to determine θ_1, e.g. $\tan(\theta_1/2) = \dfrac{(y/2)}{x}$

  Note: no mark for protractor method here.
- Perform experiment in a dark room, or shield apparatus.

Method of analysis (2 A-marks)

- Plot a graph of θ_1 against λ.
- Relationship is valid if graph is a straight line and passes through origin.

Safety considerations (1 S-mark)

- To prevent eye damage from bright/intense sources, wear dark glasses/do not look at source directly/shield source.
- To prevent burns from hot sources, wear gloves/switch off when not in use/avoid touching.

Additional detail (4 D-marks)

- Use a large x to obtain a large y.
- Obtain a large y to reduce percentage uncertainty in the measurement (of y and θ_1).
- Use vernier calipers to measure y.
- Repeat experiment (i.e. take many readings of y) for each λ (or pair of LED) and average.
- Method to check brightness of light sources is constant, e.g. measure current through the light sources regularly with ammeter.
- Method to ensure brightness of light sources is constant, e.g. workable circuit diagram with variable power supply and ammeter.
- Use a protective resistor (to keep current through LEDs under the safe value).
- View with the same eye.
- Method to ensure distances (x and y) are perpendicular, or observer equidistant from pair of light sources.
- Use vertical filament lamps/vertical slits.
- Additional detail on measuring λ, e.g. use of equation for Young's slit/diffraction grating method.
- For small angles: $\theta = \sin\theta = \tan\theta$.
- Further safety consideration (from the ones stated under safety considerations' section).

Sample Question 1.13 *(P51/O/N/09)*

The volume of air in a bottle affects its resonant frequency. It is suggested that the resonant frequency f is related to the volume V by the equation:

$$f^2 = \frac{k}{V}$$

where k is a constant. Design a laboratory experiment to determine whether this equation is correct. You should draw a diagram showing the arrangement of your equipment. In your account you should pay particular attention to:

(a) the procedure to be followed,

(b) the measurements to be taken,

(c) the control of variables,

(d) how to analyse the data,

(e) the safety precautions to be taken. [15]

Solution

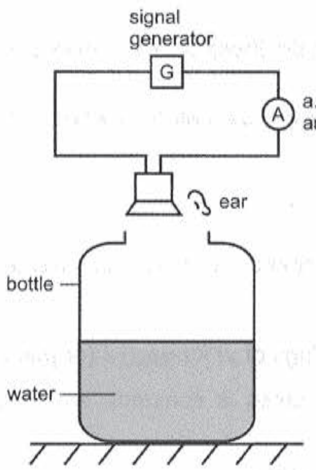


Figure 1.13.1

Defining the problem (3 P-marks)

- V is the independent variable, or vary V (e.g. by adding water to bottle).
- f is the dependent variable, or measure f (for each value of V).
- Keep temperature constant.

Methods of data collection (5 M-marks)

- Labelled diagram including source of sound adjacent to the opening, e.g. loudspeaker/tuning fork.
- Method of producing sound of different frequencies, e.g. use signal generator/variable frequency a.c. power supply/several tuning forks.
- Method of measuring volume of air, e.g. volume of container – volume of water, or find total volume of each different container.
- Method of determining resonant frequency, e.g. when loudest sound is heard.
- Perform experiment in a quiet room, or avoid other noise.

Method of analysis (2 A-marks)

- Plot a graph of f^2 against $1/V$.
- Relationship is correct if graph is a straight line and passes through origin.

Safety considerations (1 S-mark)

- To prevent pain/ear damage from loud speaker, wear ear defenders/switch off when not in use.

Additional detail (4 D-marks)

- Detail on measuring volume, e.g. use of measuring cylinder/burette.
- Detail on determining resonance, e.g. first determine resonant frequency by increasing the frequency of signal generator, then by decreasing it and then average, or determine resonant frequency by adding/subtracting small amounts of water.
- Every time hear with the same ear.
- Determination of frequency, e.g. read off signal generator/tuning fork/use c.r.o.
- Keep loudness/amplitude/intensity/power output of source of sound/(rms) current through it constant.
- Method to check loudness of source of sound is constant, e.g. measure (rms) current through the loudspeaker regularly with ammeter.
- Use a.c. ammeter.
- Method to ensure loudness of source of sound is constant, e.g. adjust signal generator to keep (rms) current constant.
- Gradient = k.
- Discussion of container, e.g. end correction/shape of mouth of bottle.
- Method to check fundamental frequency.
- Detail on determining period, and hence frequency, from c.r.o., i.e. period = length × time-base and frequency = 1/period.

Sample Question 1.14 *(P05/O/N/07)*

Double glazing can be used for sound insulation. Double-glazed windows consist of two panes of glass with air in the space between them. Manufacturers reduce the air pressure in the space between the panes of glass to reduce the amplitude of sound transmitted through the window. It is suggested that the amplitude A of sound transmitted through a double-glazed window is related to the air pressure p in the space between the panes by the equation:

$$A = k\sqrt{p}$$

where k is a constant. Fig. 1.14.1 shows a laboratory model of a double-glazed window. It consists of two panes of glass. There is a tube connected to the space between the two panes so that air may be removed.

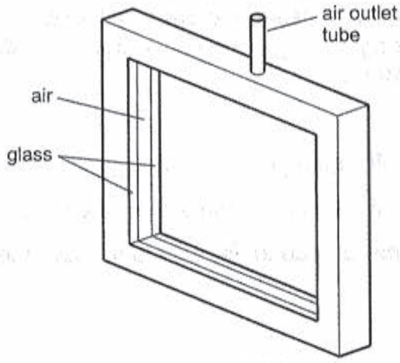


Figure 1.14.1

Design a laboratory experiment to investigate whether A is related to p as indicated in the above equation when p is reduced. You should draw a diagram showing the arrangement of your equipment. In your account you should pay particular attention to:

(a) the procedure to be followed,

(b) how the air pressure between the panes of glass would be measured,

(c) how the amplitude of the sound would be measured,

(d) the control of variables,

(e) how the data would be analysed,

(f) any safety precautions that you would take. [15]

Solution

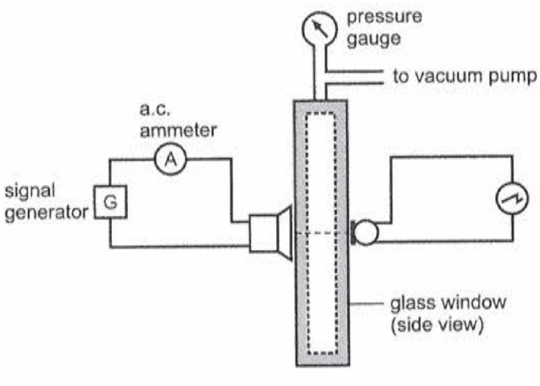


Figure 1.14.2

Defining the problem (3 P-marks)

- *p* is the independent variable, or vary *p*.
- *A* is the dependent variable, or determine *A* (for each value of *p*).
- Keep incident amplitude/loudness/intensity/power output of source of sound/(rms) current through it constant.

Methods of data collection (5 M-marks)

- Labelled diagram of a workable arrangement including source of sound, glass window and detector of sound.
  Allowed sources: loudspeaker, bell, buzzer, siren but **not** musical instrument.
  Allowed detectors: microphone, sound meter, sound detector.
- Method of measuring pressure, e.g. use pressure gauge/manometer/bourdon gauge.
  Note: no mark for barometer or pressure meter.
- Method of reducing pressure, e.g. use (vacuum) pump to withdraw air from glass window.
- Method of measuring amplitude, e.g. use c.r.o. to measure amplitude.
- Perform experiment in a quiet room.

Method of analysis (2 A-marks)

- Plot a graph of A^2 against *p*.
- Relationship is correct if graph is a straight line and passes through origin.

Safety considerations (1 S-mark)

- To prevent pain/ear damage from loud sound, wear ear defenders/switch off when not in use.
- To prevent injury in case glass breaks (due to low air pressure inside), wear goggles/use safety screen.

Additional detail (4 D-marks)

- Window perpendicular to source of sound.
- Method to reduce the effect of sound reflection, e.g. speaker and microphone close to glass/use of foam.
- Use a loud incident source to obtain a large amplitude at small pressures.
- Obtain a large amplitude to reduce percentage uncertainty in its measurement.
- Method to check incident amplitude/loudness of source of sound is constant, e.g. measure (rms) current through the loudspeaker regularly with ammeter.
- Use a.c. ammeter.
- Method to ensure incident amplitude/loudness of source of sound is constant, e.g. adjust signal generator to keep (rms) current constant.
- Control (or monitoring) of one additional variable, e.g. distances between loudspeaker, glass and microphone, or temperature, or frequency.
- Wait for pressure between the glass/temperature to stabilise.
- Gradient = k^2.
- Detail on determining amplitude from c.r.o., e.g. amplitude = height × *y*-gain.
- Further safety consideration (from the ones stated under safety considerations' section).

Sample Question 1.15 *(P52/M/J/12)*

A hot air balloon is tied to the ground using a rope. As the wind blows with speed *v*, the rope makes an angle θ to the horizontal, as shown in Fig. 1.15.1.

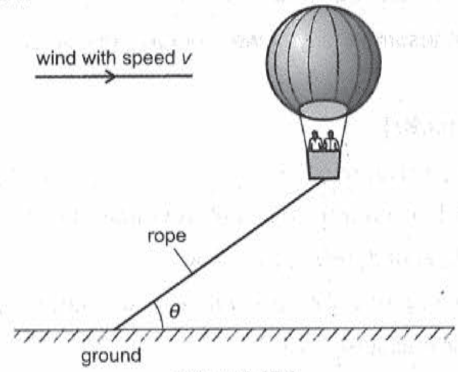


Figure 1.15.1

It is suggested that tan θ is inversely proportional to v^2. To model the hot air balloon in the laboratory, a balloon filled with helium is used. Design a laboratory experiment using a small helium-filled balloon to test the relationship between θ and *v*. You should draw a diagram showing the arrangement of your equipment. In your account you should pay particular attention to:

(a) the procedure to be followed,

(b) the measurements to be taken,

(c) the control of variables,

(d) the analysis of the data,

(e) the safety precautions to be taken. [15]

Solution

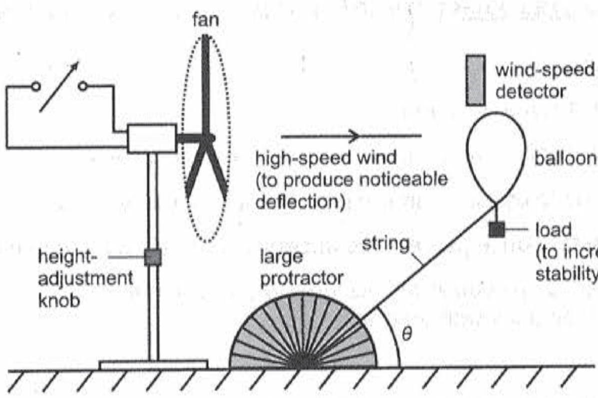


Figure 1.15.2

Defining the problem (3 P-marks)

- *v* is the independent variable <u>and</u> θ is the dependent variable, or vary *v* <u>and</u> measure θ (for each value of *v*).
- Keep the (shape <u>and</u>) size/volume/surface area/mass of balloon/helium <u>constant</u>.

  (so that the upthrust/weight remains constant).

  Note: no mark for stating: 'same balloon'.
- Keep the temperature (of air/helium/balloon) <u>constant</u>.

  (so that the volume remains constant).

Methods of data collection (5 M-marks)

- Labelled diagram of apparatus: balloon, string fixed and method of producing wind.
  Method of producing wind should be approximately horizontal to balloon.
- Suspend mass from balloon.
- Method to change wind speed, e.g. use variable power supply/variable resistor/change distance from fan.
- Method to measure wind speed, e.g. use wind-speed detector/indicator/anemometer.
- Method to measure angle θ, e.g. use protractor/rule for measurements for trigonometry methods.

Method of analysis (2 A-marks)

- Plot a graph of tan θ against $1/v^2$.
- Relationship is correct if graph is a straight line and passes through origin.

Safety considerations (1 S-mark)

- To prevent injury from moving blades of fan, use a fan with protective grill/switch off when changing setting/avoid touching.
- To prevent air stream entering eyes, wear goggles.

Additional detail (4 D-marks)

- Keep length of the thread constant.
- Method to produce noticeable deflection, e.g. use high-speed wind/large cross-sectional area of balloon/small length of thread.
- Reason for adding mass, e.g. to increase stability/deflection.
- Wait for balloon to become stable.
- Keep windows shut/air conditioning switched off/use of wind tunnel to avoid draughts.
- Additional detail on measuring angle θ, e.g. use a large protractor (to reduce percentage uncertainty in the measurement of θ)/projection method.
  (A large protractor has smaller least count than normal; so it gives smaller percentage uncertainty.)
- Avoid parallax error when measuring angle θ with protractor.
- Measuring air speed at point where balloon is positioned.
- Adjust height of fan so that air flow is horizontally aligned to the balloon.
- tan θ = height / base (for trigonometry method).
- Further safety consideration (from the ones stated under safety considerations' section).

Sample Question 1.16 *(P51/M/J/12)*

A fairground ride carries passengers in chairs which are attached by metal rods to a rotating central pole, as shown in Fig. 1.16.1. When the pole rotates with angular velocity ω, the rods make an angle θ to the vertical.

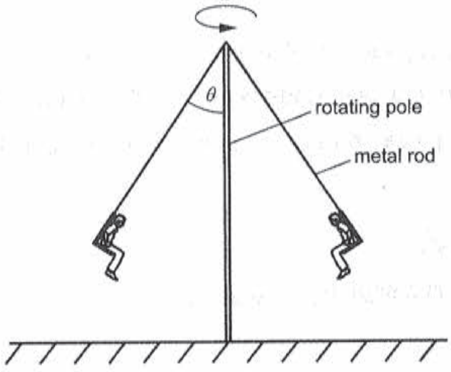


Figure 1.16.1

It is suggested that cos θ is inversely proportional to ω^2. Design a laboratory experiment, using a small object to represent an occupied chair, to test the relationship between θ and ω. You should draw a diagram showing the arrangement of your equipment. In your account you should pay particular attention to:

(a) the procedure to be followed,

(b) the measurements to be taken,

(c) the control of variables,

(d) the analysis of the data,

(e) the safety precautions to be taken. [15]

Solution

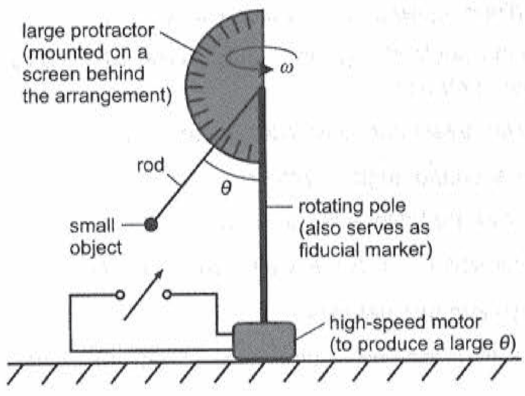


Figure 1.16.2

Defining the problem (3 P-marks)

- ω or frequency or period of rotation is the independent variable and θ is the dependent variable, or vary ω or frequency or period of rotation and measure θ (for each value of ω).
- $\omega = 2\pi f = 2\pi/T$.
- Keep length of the rigid rod constant.

Methods of data collection (5 M-marks)

- Labelled diagram of apparatus: small object, pole attached to a rotating device, e.g. motor or turntable.
- Method to change the speed of the rotating device, e.g. use variable power supply/variable resistor.
- Method to determine frequency or period of rotation, e.g. stopwatch to time a number of oscillations, light gates connected to a timer/frequency meter.
- Use fiducial marker, or light gates perpendicular to motion of object.
- Method to measure angle θ, e.g. use protractor/rule for measurements for trigonometry methods.

Method of analysis (2 A-marks)

- Plot a graph of cos θ against $1/\omega^2$.
- Relationship is correct if graph is a straight line and passes through origin.

Safety considerations (1 S-mark)

- To prevent injury in case mass detaches from the pole or rod, use a protective screen.

Additional detail (4 D-marks)

- Method of checking pole is vertical, e.g. use a spirit level/set square.
- Use high-speed motor to produce a large θ.
- Produce a large θ to reduce percentage uncertainty in its measurement.
- Wait for motion to become stable.
- Additional detail on measuring angle θ, e.g. use a large protractor (to reduce percentage uncertainty in the measurement of θ)/projection method.
- Avoid parallax error when measuring angle θ with protractor.
- Additional detail on measuring angular velocity, e.g. time at least 10 rotations.
- Take many readings of time (to determine frequency/angular velocity) for each θ and average.
- Projection method, slow motion freeze frame video, camera with detail.
- cos θ = height / hypotenuse or equivalent (for trigonometry method).

Sample Question 1.17 *(P51/O/N/14)*

A student investigates the power dissipated by a lamp connected to a model wind turbine as shown in Fig. 1.17.1.

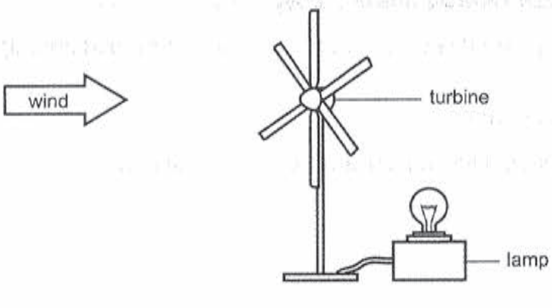


Figure 1.17.1

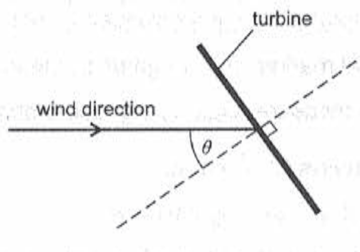


Figure 1.17.2

The power P dissipated in the lamp depends on the angle θ between the axis of the turbine and the direction of the wind, as shown by the top view in Fig. 1.17.2. It is suggested that:

$$P = k \cos \theta$$

where k is a constant. Design a laboratory experiment to test the relationship between P and θ and determine a value for k. You should draw a diagram showing the arrangement of your equipment. In your account, you should pay particular attention to:

(a) the procedure to be followed,

(b) the measurements to be taken,

(c) the control of variables,

(d) the analysis of the data,

(e) the safety precautions to be taken. [15]

Solution

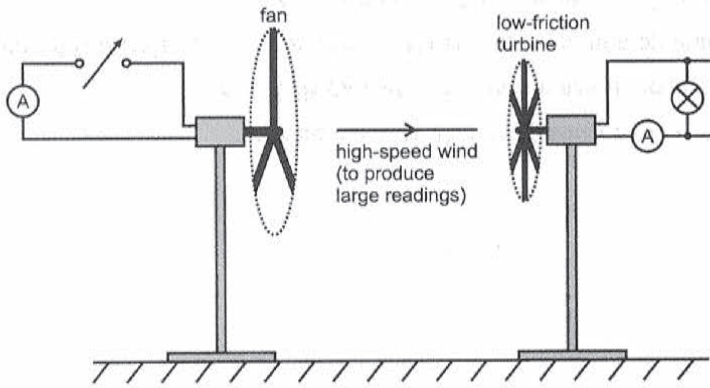


Figure 1.17.3

Defining the problem (3 P-marks)

- θ is the independent variable, or vary θ.
- P is the dependent variable, or determine P (for each value of θ).
- Keep speed of the air/output power of the fan/current through it/voltage across it <u>constant</u>.

Methods of data collection (5 M-marks)

- Labelled diagram showing method to produce air flow in line with turbine. Method of producing wind must be labelled.
- Circuit connecting turbine to lamp with ammeter and voltmeter connected correctly.

  Note: no mark for showing additional power supply in the lamp circuit.
- Method to determine power, e.g. $P = IV$.
- Method to measure angle θ, e.g. use protractor/rule for measurements for trigonometry methods.
- Ensure that there are no other draughts or airflows.

Method of analysis (2 A-marks)

- Plot a graph of P against $\cos\theta$.
- k = gradient.

Safety considerations (1 S-mark)

- To prevent injury from moving blades of fan, use a fan with protective grill/switch off when changing setting/avoid touching.
- To prevent air stream entering eyes, wear goggles.

Additional detail (4 D-marks)

- Keep distance from fan to turbine constant.
- Method to check wind speed is constant, e.g. measure current through the fan regularly with ammeter.
- Method to ensure wind speed is constant, e.g. workable circuit diagram with variable power supply and ammeter.
- Use of low-friction turbine/low-resistance lamp.
- Use high-speed wind to produce large (i.e. measurable) readings (on ammeter and voltmeter).
- Produce large readings to reduce percentage uncertainty in the measurement of P.
- Wait for airflow/turbine/meter readings to stabilise.
- Avoid turbulence or reflection of air flow.
- Additional detail on measuring angle θ, e.g. use a large protractor (to reduce percentage uncertainty in the measurement of θ).
- Avoid parallax error when measuring angle θ with protractor.
- Relationship is correct if graph is a straight line and passes through origin (provided plotted graph is correct).
- $\cos\theta$ = height / hypotenuse or equivalent (for trigonometry method).
- Further safety consideration (from the ones stated under safety considerations' section).

Sample Question 1.18 *(P51/M/J/14)*

A ball rolls forwards and backwards on a curved track as shown in Fig. 1.18.1.

Figure 1.18.1

It is suggested that the period T of the oscillations is related to the radius r of the ball and the radius of curvature C of the track by the relationship:

$$T^2 = \frac{28\pi^2}{5g}(C - r)$$

where g is the acceleration of free fall. You are provided with a flexible track. Design a laboratory experiment to test the relationship between T and r. Explain how your results could be used to determine a value for C. You should draw a diagram showing the arrangement of your equipment. In your account you should pay particular attention to:

(a) the procedure to be followed,

(b) the measurements to be taken,

(c) the control of variables,

(d) the analysis of the data,

(e) the safety precautions to be taken. [15]

Solution

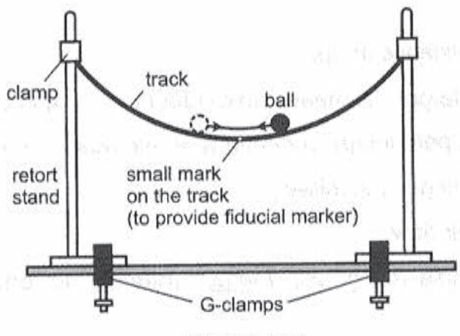


Figure 1.18.2

Defining the problem (3 P-marks)

- r is the independent variable, or vary r (i.e. use several balls of different radii).
- T is the dependent variable, or measure T (for each value of r).
- Keep C constant.

  Note: no mark for stating: 'use same track'.

Methods of data collection (5 M-marks)

- Labelled diagram showing ball in a (curved) track with supports for track, e.g. retort stands.

  Note: Minimum of two labels from: ball, track, supports; **not** stopwatch, bench, micrometer. Also, supports should be making contact with track higher than ball (at least half way up).
- Measure time using stopwatch, or light gates connected to a timer.
- Time many oscillations (at least 10 or at least 10 s of timing) and determine period using $T = t / n$.
- Measure diameter (or radius) of ball with micrometer/vernier calipers.
- radius = diameter / 2.

Method of analysis (2 A-marks)

- Plot a graph of T^2 against r.
- $C = y - \text{intercept} \times \dfrac{5g}{28\pi^2}$.

Safety considerations (1 S-mark)

- To prevent (injury from) balls rolling on to floor, use safety screen/barrier/sand tray.

Additional detail (4 D-marks)

- Clean track/balls.

  Note: no mark for stating: 'oil the track'.
- G-clamp retort stands/add weights to retort stands.
- Use small amplitude of oscillation (to ensure equation remains valid).
- Use of fiducial marker near center of track/mark on the track.
- Time many oscillations to reduce percentage uncertainty in the measurement (of T).
- Take many readings of t for each ball and average.
- Keep material/density of balls constant.
- Take many readings of diameter in different directions for each ball and average.
- Relationship is correct if graph is a straight line, (provided plotted graph is correct).
- Relationship is correct if graph is a straight line not passing through origin or has an intercept (provided plotted graph is correct).

Sample Question 1.19 *(P52/M/J/10)*

A current in a flat circular coil produces a magnetic field. A student suggests that the strength B of the magnetic field is related to the distance x from the centre of the coil (see Fig. 1.19.1) by the equation:

$$B = B_0 e^{-px}$$

where B_0 is the strength of the magnetic field for $x = 0$, and p is a constant.

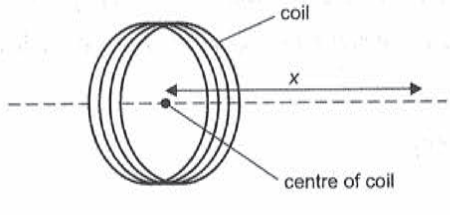


Figure 1.19.1

Design a laboratory experiment that uses a Hall probe to investigate the relationship between B and x. You should draw a diagram showing the arrangement of your equipment. In your account you should pay particular attention to:

(a) the procedure to be followed,

(b) the measurements to be taken,

(c) the control of variables,

(d) the analysis of the data,

(e) the safety precautions to be taken. [15]

Solution

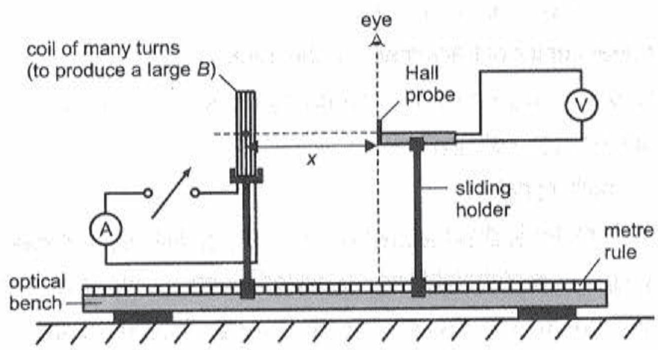


Figure 1.19.2

Defining the problem (3 P-marks)

- x is the independent variable <u>and</u> B is the dependent variable, or vary x <u>and</u> measure B (for each value of x).
- Keep the number of turns on the coil/radius of the coil <u>constant</u>.

  Note: no mark for stating: 'same coil'.
- Keep the <u>current</u> in the coil <u>constant</u>.

Methods of data collection (5 M-marks)

- Labelled diagram showing flat coil and Hall probe with a means of read out appropriately positioned along axis.

  Note: no mark for labelling the coil as 'solenoid'.
- Circuit diagram for coil connected to a (d.c.) power supply.

  Note: no mark for a.c. power supply.
- Measure *x* with ruler.
- Keep Hall probe at right angles to direction of magnetic field, or adjust Hall probe to obtain maximum (voltmeter) reading for each *x*.
- Method to locate $x = 0$ position, e.g. adjust Hall probe to obtain overall maximum (voltmeter) reading/use cross rule.

Method of analysis (2 A-marks)

- Plot a graph of ln *B* against *x*.
- Relationship is correct if graph is a straight line.

Safety considerations (1 S-mark)

- To prevent burns from hot coil, wear gloves/switch off when not in use/avoid touching.

Additional detail (4 D-marks)

- Method to produce a large *B*, e.g. use large number of turns /large current.
- Produce a large *B* to reduce percentage uncertainty in its measurement.
- Method to check current in the coil is constant, e.g. measure current through coil regularly with ammeter.
- Method to ensure current in the coil is constant, e.g. workable circuit diagram with variable (d.c.) power supply or variable resistor.
- Method to keep Hall probe in same orientation, e.g. use optical bench, or use set square, or fix to rule.
- Method to keep Hall probe along axis, e.g. use of sliding holder.
- *B* is proportional to voltage across Hall probe/calibrate Hall probe (in a known magnetic field).
- Repeat experiment with Hall probe reversed and find average values (of magnitudes) of *B* for each *x*.
- Avoid external magnetic fields.
- Avoid parallax error when measuring *x*.
- $\ln B = -px + \ln B_0$.

Sample Question 1.20 *(P51/O/N/11)*

A current-carrying coil produces a magnetic field. It is suggested that the strength B of the magnetic field at the centre of a flat circular coil is inversely proportional to the radius r of the coil. Design a laboratory experiment that uses a Hall probe to test the relationship between B and r. You should draw a diagram showing the arrangement of your equipment. In your account you should pay particular attention to:

(a) the procedure to be followed,

(b) the measurements to be taken,

(c) the control of variables,

(d) the analysis of the data,

(e) the safety precautions to be taken. [15]

Solution

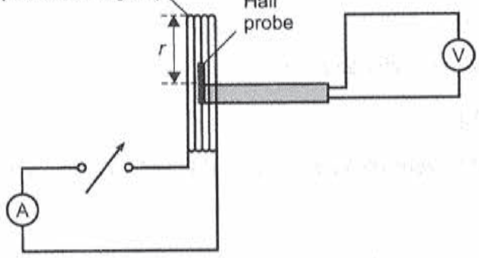


Figure 1.20.1

Defining the problem (3 P-marks)

- r is the independent variable <u>and</u> B is the dependent variable, or vary r (i.e. use several coils of different radii) <u>and</u> measure B (for each value of r).
- Keep the number of turns on the coil(s) <u>constant</u>.
  Note: no mark for stating: 'same coil'.
- Keep the <u>current</u> in the coil(s) <u>constant</u>.

Methods of data collection (5 M-marks)

- <u>Labelled</u> diagram showing flat coil and Hall probe positioned in the centre of the coil.
- Circuit diagram for coil connected to a (d.c.) power supply.
- Connect Hall probe to voltmeter/c.r.o.
- Measure diameter (or radius) with ruler/vernier calipers.
  Note: no mark for 'micrometer (screw gauge)'.
- Method to locate centre of coil, e.g. adjust Hall probe to obtain maximum (voltmeter) reading/use cross rule.

Method of analysis (2 A-marks)

- Plot a graph of B against $1/r$.
- <u>Relationship is valid if</u> graph is a <u>straight line and passes through origin</u>.

Safety considerations (1 S-mark)

- To prevent burns from hot coil, wear gloves/switch off when not in use/avoid touching.

Additional detail (4 D-marks)

- Take many readings of diameter (or radius) in different directions for each coil and average.
- Method to produce a large B, e.g. use large number of turns/large current.
- Produce a large B to reduce percentage uncertainty in its measurement.
- Method to check current in the coil(s) is constant, e.g. measure current through coil regularly with ammeter.
- Method to ensure current in the coil(s) is constant, e.g. workable circuit diagram with variable (d.c.) power supply or variable resistor.
- Keep Hall probe at right angles to direction of magnetic field (at the centre of the coil)
- Method to keep Hall probe in same orientation, e.g. use optical bench, or use set square, or fix to rule.
- B is proportional to voltage across Hall probe/calibrate Hall probe (in a known magnetic field).
- Repeat experiment with Hall probe reversed and find average values (of magnitudes) of B for each r (or coil).
- Avoid external magnetic fields.

Sample Question 1.21 *(P52/M/J/14)*

Two identical coils are connected together and arranged as shown in Fig. 1.21.1.

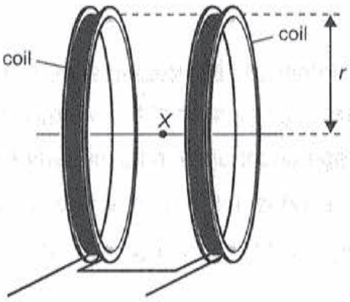


Figure 1.21.1

The coils are in the vertical plane and are parallel to each other. When the coils are connected to a power supply, there is a magnetic field between them. It is suggested that the magnetic flux density *B* of the field at the point X is related to the radius *r* of the coils by the relationship:

$$B = \frac{0.72\mu_0 NI}{r}$$

where *N* is the number of turns on each coil, *I* is the current in the coils and μ_0 is the permeability of free space. Design a laboratory experiment that uses a Hall probe to test the relationship between *B* and *r* and determine a value for μ_0. You should draw a diagram showing the arrangement of your equipment. In your account you should pay particular attention to:

(a) the procedure to be followed,

(b) the measurements to be taken,

(c) the control of variables,

(d) the analysis of the data,

(e) the safety precautions to be taken. [15]

Solution

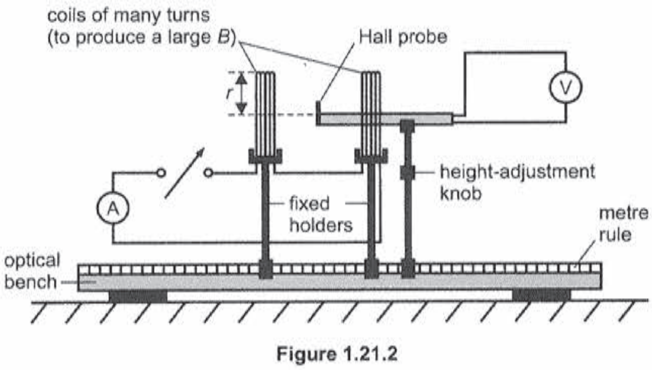


Figure 1.21.2

Defining the problem (3 P-marks)

- *r* is the independent variable <u>and</u> *B* is the dependent variable, or vary *r* (i.e. use several coil pairs of different radii) <u>and</u> measure *B* (for each value of *r*).
- Keep the number of turns on the coil(s) <u>constant</u>.

  Note: no mark for stating: 'same coil'.
- Keep the <u>current</u> in the coil(s) <u>constant</u>.

Methods of data collection (5 M-marks)

- Labelled diagram showing flat coils and Hall probe positioned at X.
- Workable Circuit diagram for coils connected to a (d.c.) power supply and ammeter.
- Connect Hall probe to voltmeter/c.r.o.
- Measure diameter (or radius) with ruler/vernier calipers.
- Calibrate Hall probe (in a known magnetic field).

Method of analysis (2 A-marks)

- Plot a graph of B against $1/r$.
- $\mu_0 = \frac{\text{gradient}}{0.72NI}$.

Safety considerations (1 S-mark)

- To prevent burns from hot coil, wear gloves/switch off when not in use/avoid touching.

Additional detail (4 D-marks)

- Take many readings of diameter (or radius) in different directions for each pair of coils and average.
- Method to produce a large B, e.g. use large number of turns/large current.
- Produce a large B to reduce percentage uncertainty in its measurement.
- Method to ensure current in the coil(s) is constant, e.g. workable circuit diagram with variable (d.c.) power supplyor variable resistor.
- Method to locate X, e.g. adjust Hall probe to obtain maximum (voltmeter) reading.
- Method to keep Hall probe perpendicular to direction of magnetic field at X, e.g. use optical bench,or use set square, or fix to rule.
- Repeat experiment with Hall probe reversed and find average values (of magnitudes) of B for each r (or pair of coils).
- Avoid external magnetic fields.
- Keep the distance between coils constant.
- Method to keep the distance between coils constant, e.g. fix holders.
- Method to check coils are correctly aligned in parallel.
- Relationship is valid if graph is a straight line and passes through origin (provided plotted graph is correct).

Sample Question 1.22 *(P51/O/N/12)*

As a bar magnet is dropped through a coil, an e.m.f. is induced in the coil. The maximum e.m.f. *E* is induced as the magnet leaves the coil with speed *v*. It is suggested that *E* is directly proportional to *v*. Design a laboratory experiment to test the relationship between *E* and *v*. You should draw a diagram showing the arrangement of your equipment. In your account you should pay particular attention to:

(a) the procedure to be followed,

(b) the measurements to be taken,

(c) the control of variables,

(d) the analysis of the data,

(e) the safety precautions to be taken. [15]

Solution

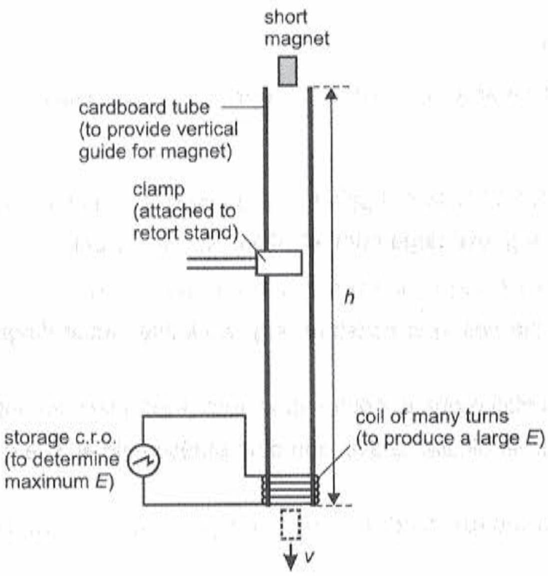


Figure 1.22.1

Defining the problem (3 P-marks)

- *v* is the independent variable, or vary *v*.
- *E* is the dependent variable, or measure *E* (for each value of *v*).
- Keep the number of turns on the coil constant.

Methods of data collection (5 M-marks)

- Labelled diagram showing magnet falling vertically through coil.
- C.R.O. or voltmeter connected to the coil.
- Method to change *v*, e.g. change height *h* (i.e. cut the vertical guide).
- Measurements to determine *v*, e.g. use metre rule to measure distance (*h*) the magnet falls to the bottom of the coil.
- Method of determining *v*, e.g. $v = \sqrt{2gh}$.

Method of analysis (2 A-marks)

- Plot a graph of E against v.
- Relationship is valid if graph is a straight line and passes through origin.

Safety considerations (1 S-mark)

- To prevent injury from falling magnet, keep hands and feet away/use sand tray or cushion to catch magnet.

Additional detail (4 D-marks)

- Use a non-metallic vertical guide/tube.
- Method to support vertical coil or guide/tube.
- Method to ensure coil or guide/tube is vertical, e.g. use spirit level/set square.
- Method to produce a large E, e.g. use large number of turns on coil/drop magnet from large heights/use strong magnet.
- Produce a large E to reduce percentage uncertainty in its measurement.
- Detail on determining E, e.g. use storage c.r.o. to determine maximum E/use video camera including slow motion playback.
- Detail on determining E from c.r.o., e.g. E = height × y-gain.
- Take many readings of E for each v (or h) and average.
- Use same magnet/magnet of same strength.
- Use short magnet/thin coil so that v is (nearly) constant.

Sample Question 1.23 *(P53/O/N/14)*

A thin card is inserted between two separate iron cores. A coil is wound around one core as shown in Fig. 1.23.1.

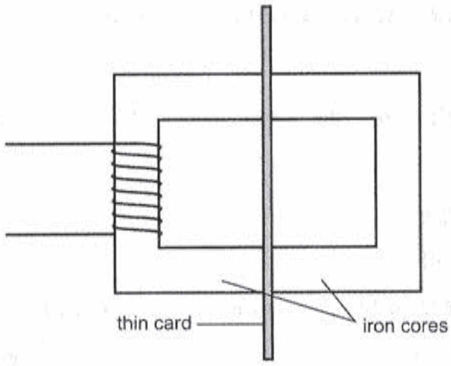


Figure 1.23.1

A current in the coil may induce an e.m.f. in another coil wound on the other core. The induced e.m.f. V depends on the thickness t of the card. A student suggests that:

$$V = V_0 e^{-\sigma t}$$

where V_0 is the induced e.m.f. without card between the cores and σ is a constant. Design a laboratory experiment to test the relationship between V and t and determine the value of σ. You should draw a diagram showing the arrangement of your equipment. In your account you should pay particular attention to:

(a) the procedure to be followed,

(b) the measurements to be taken,

(c) the control of variables,

(d) the analysis of the data,

(e) the safety precautions to be taken. [15]

Solution

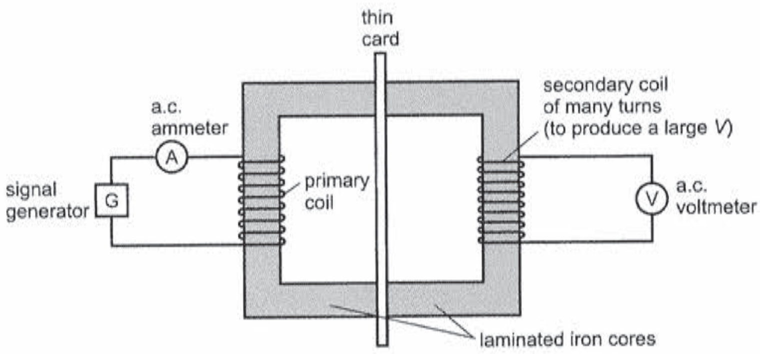


Figure 1.23.2

Defining the problem (3 P-marks)

- t is the independent variable, or vary t (i.e. use several cards of different thicknesses).
- V is the dependent variable, or measure V (for each value of t).
- Keep the (rms) current (in the primary coil) constant.

Methods of data collection (5 M-marks)

- Labelled diagram showing two independent coils wound on iron cores.
- Signal generator/a.c. power supply connected to one coil in a workable circuit.
- Voltmeter/c.r.o. connected to other coil in a workable circuit.
- Measure thickness of card with micrometer/vernier calipers.
- Method to keep (rms) current in the primary coil constant, e.g. adjust signal generator/use of rheostat.

Method of analysis (2 A-marks)

- Plot a graph of ln V against t.
- $\sigma = -$ gradient

Safety considerations (1 S-mark)

- To prevent burns from hot coil(s), wear gloves/switch off when not in use/avoid touching.

Additional detail (4 D-marks)

- Use laminated cores or use insulated wire for coils.
- Measurement of V_0 stating that no card is present.
- Take many readings of thickness t at different points for each card and average.
- Discussion of compression of card, e.g. measure t when card is secured.
- Method to produce a large V, e.g. use large number of turns on secondary coil/use large current in primary coil/use high frequency a.c.

  (as by Faraday's law, e.m.f. induced in a coil is directly proportional to the rate of change of magnetic flux linking **that** coil.)
- Produce a large V to reduce percentage uncertainty in its measurement.
- Use ammeter to check (rms) current in the primary coil is constant.
- Use a.c. ammeter (and a.c. voltmeter).
- Keep frequency of a.c. source constant or keep the number of turns on (each) coil constant.
- $\ln V = \ln V_0 - \sigma t$.
- Relationship is valid if graph is a straight line with y-intercept = $\ln V_0$ (provided plotted graph is correct).
- Detail on determining induced e.m.f. V from c.r.o., e.g. peak value of V = height × y-gain.

Sample Question 1.24 *(P51/O/N/10)*

Fig. 1.24.1 shows a coil (coil X).

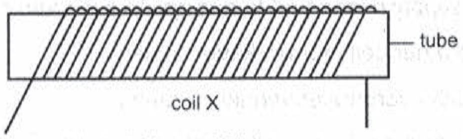


Figure 1.24.1

A student winds another coil (coil Y) tightly around coil X. A changing e.m.f. in coil X induces an e.m.f. in coil Y. The student wishes to investigate how the e.m.f. *V* in coil Y depends on the frequency *f* of the current in coil X. It is suggested that *V* is directly proportional to *f*. Design a laboratory experiment to investigate the suggested relationship. You should draw a diagram showing the arrangement of your equipment. In your account you should pay particular attention to:

(a) the procedure to be followed,

(b) the measurements to be taken,

(c) the control of variables,

(d) the analysis of the data,

(e) the safety precautions to be taken. [15]

Solution

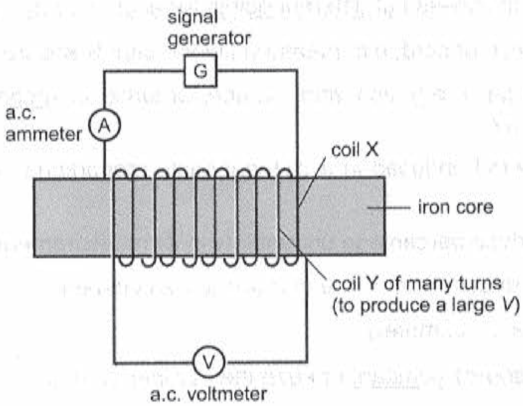


Figure 1.24.2

Defining the problem (3 P-marks)

- *f* is the independent variable and *V* is the dependent variable, or vary *f* and measure *V* (for each value of *f*).
- Keep the (rms) current in coil X constant.
- Keep the number of turns on (each) coil/area of coil Y constant.

Methods of data collection (5 M-marks)

- Labelled diagram showing two independent coils X and Y wound on a tube (or iron core).
- Signal generator/variable frequency a.c. power supply connected to coil X in a workable circuit.
- Voltmeter/c.r.o. connected to coil Y in a workable circuit.
- Method to determine frequency *f*, e.g. read off signal generator/use c.r.o.
- Method to keep (rms) current in coil X constant, e.g. adjust signal generator/use of rheostat.

Theory: When the frequency of a.c. flowing in a circuit containing a coil (and/or a capacitor) is changed, the overall resistance of the circuit (which is known as impedance) also changes, and so does the (rms) current flowing in it.

Method of analysis (2 A-marks)

- Plot a graph of *V* against *f*.
- Relationship is valid if graph is a straight line and passes through origin.

Safety considerations (1 S-mark)

- To prevent burns from hot coil(s), wear gloves/switch off when not in use/avoid touching.

Additional detail (4 D-marks)

- Use insulated wire for coils.
- Method to produce a large *V*, e.g. use large number of turns on coil Y/use large current in coil X/use high frequency a.c.
- Use iron core (to produce a large *V*).
- Produce a large *V* to reduce percentage uncertainty in its measurement.
- Use ammeter to check (rms) current in coil X is constant.
- Use a.c. ammeter (and a.c. voltmeter).
- Keep coil Y and coil X in the same relative positions.
- Avoid other alternating magnetic fields.
- Detail on determining induced e.m.f. *V* from c.r.o., e.g. peak value of V = height × *y*-gain.
- Detail on determining period, and hence frequency, from c.r.o., e.g. period = length × time-base and frequency = 1/period.

Sample Question 1.25 *(P53/O/N/11)*

A changing e.m.f. in a coil can induce an e.m.f. in another coil. Fig. 1.25.1 shows a coil (coil X), which is wound on a cardboard tube.

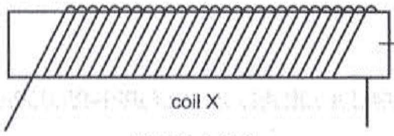


Figure 1.25.1

Coil X has cross-sectional area *A*. A student winds another coil (coil Y) tightly around coil X. The student wishes to investigate how the e.m.f. *V* in coil Y depends on *A*. It is suggested that *V* is directly proportional to *A*. Design a laboratory experiment to investigate the suggested relationship. You should draw a diagram showing the arrangement of your equipment. In your account you should pay particular attention to:

(a) the procedure to be followed,

(b) the measurements to be taken,

(c) the control of variables,

(d) the analysis of the data,

(e) the safety precautions to be taken. [15]

Solution

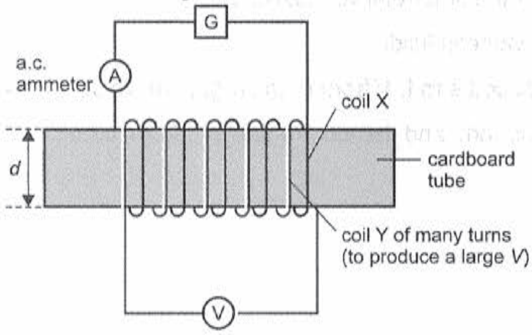


Figure 1.25.2

Defining the problem (3 P-marks)

- *A* is the independent variable and *V* is the dependent variable, or vary *A* (i.e. use several coils of different cross-sectional areas) and measure *V* (for each value of *A*).
- Keep the (rms) current in coil X constant.
- Keep the number of turns on (each) coil constant.

Methods of data collection (5 M-marks)

- Labelled diagram showing two independent coils X and Y wound on a tube.
- Signal generator/a.c. power supply connected to coil X in a workable circuit.
- Voltmeter/c.r.o. connected to coil Y in a workable circuit.
- Measure diameter/radius with ruler/vernier calipers.
- Method to determine area *A*, e.g. $A = \pi r^2$.

Method of analysis (2 A-marks)

- Plot a graph of *V* against *A*.
- Relationship is valid if graph is a straight line and passes through origin.

Safety considerations (1 S-mark)

- To prevent burns from hot coil(s), wear gloves/switch off when not in use/avoid touching.

Additional detail (4 D-marks)

- Use insulated wire for coils.
- Take many readings of diameter *d* in different directions for each coil (pair) and average.
- Method to produce a large *V*, e.g. use large number of turns on coil Y/use large current in coil X/use high frequency a.c.
- Produce a large *V* to reduce percentage uncertainty in its measurement.
- Use ammeter to check (rms) current in coil X is constant.
- Use a.c. ammeter (and a.c. voltmeter).
- Method to keep (rms) current in coil X constant, e.g. adjust signal generator/use of rheostat.
- Keep frequency of a.c. source constant.
- Keep coil Y and coil X in the same relative positions.
- Avoid other alternating magnetic fields.
- Detail on determining induced e.m.f. *V* from c.r.o., e.g. peak value of V = height × *y*-gain.

Sample Question 1.26 *(P51/O/N/13)*

An aluminium ring is placed on a coil with the rod of a metal retort stand passing through their centres, as shown in Fig. 1.26.1.

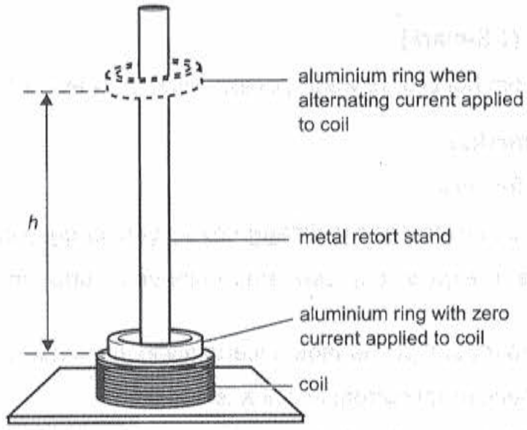


Figure 1.26.1

When an alternating current of frequency f is applied to the coil, the ring rises until it is in equilibrium at a height h above the coil. It is suggested that the relationship between h and f is:

$$h = kf^n$$

where k and n are constants. Design a laboratory experiment to test the relationship between h and f and determine values for k and n. You should draw a diagram showing the arrangement of your equipment. In your account you should pay particular attention to:

(a) the procedure to be followed,

(b) the measurements to be taken,

(c) the control of variables,

(d) the analysis of the data,

(e) the safety precautions to be taken. [15]

Solution

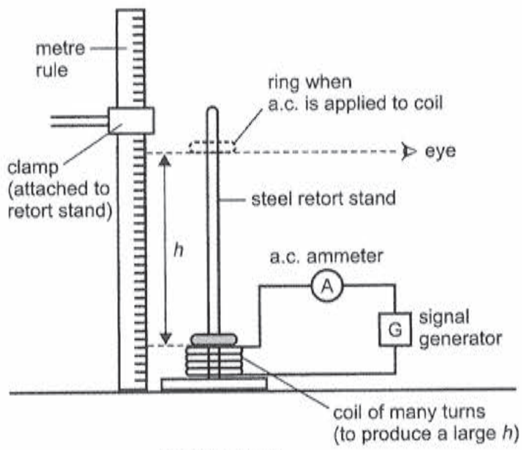


Figure 1.26.2

Defining the problem (3 P-marks)

- f is the independent variable, or vary f.
- h is the dependent variable, or measure h (for each value of f).
- Keep (rms) current in the coil constant.

Methods of data collection (5 M-marks)

- Labelled diagram showing a.c. source connected to coil in a workable circuit, stand and ring.
- Method to vary f, e.g. use of signal generator/variable frequency a.c. power supply.
- Measure h with rule/vernier calipers.
- Method to determine frequency f, e.g. read off signal generator/use c.r.o.
- Take (two) readings of h from opposite sides of the ring for each frequency and average/wait for ring to stabilise.

Method of analysis (2 A-marks)

- Plot a graph of lg h against lg f.
- n = gradient and $k = 10^{y\text{-intercept}}$.

Safety considerations (1 S-mark)

- To prevent burns from hot coil, wear gloves/switch off when not in use/avoid touching.

Additional detail (4 D-marks)

- Use steel retort stand (as it is difficult to magnetise and demagnetise).
- Use insulated wire for coil.
- Method to produce a large h, e.g. use large number of turns on coil/use large current/use high frequency a.c.
- Produce a large h to reduce percentage uncertainty in its measurement.
- Method to check rule and steel retort stand are both vertical, e.g. use a spirit level/set square.
- Method to hold rule in the upright position, e.g. use retort stand and clamp arrangement.
- Avoid parallax error when measuring h.
- Use ammeter to check (rms) current in the coil is constant.
- Use a.c. ammeter.
- Method to keep (rms) current in the coil constant, e.g. adjust signal generator/use of rheostat.
- $\lg h = n \lg f + \lg k$.
- Relationship is valid if graph is a straight line (provided plotted graph is correct).
- Detail on determining period, and hence frequency, from c.r.o., i.e. period = length × time-base and frequency = 1/period.

Sample Question 1.27 *(P52/M/J/13)*

A student is investigating how the peak alternating current I_0 varies with frequency f in a circuit containing a coil of wire. It is suggested that:

$$\left(\frac{V_0}{I_0}\right)^2 = R^2 + 4\pi^2 f^2 L^2$$

where R is the resistance of the coil, V_0 is the peak voltage and L is a constant. Design a laboratory experiment to test the relationship between I_0 and f and determine a value for L. You should draw a diagram showing the arrangement of your equipment. In your account you should pay particular attention to:

(a) the procedure to be followed,

(b) the measurements to be taken,

(c) the control of variables,

(d) the analysis of the data,

(e) the safety precautions to be taken. [15]

Solution

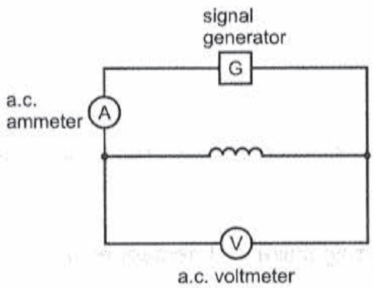


Figure 1.27.1

Defining the problem (3 P-marks)

- f is the independent variable, or vary f.
- I_0 is the dependent variable, or measure I_0 (for each value of f).
- Keep V_0 constant.

Methods of data collection (5 M-marks)

- Labelled diagram showing a.c. source connected to coil in a workable circuit.
- Method to vary f, e.g. use of signal generator/variable frequency a.c. power supply.
- Method to measure frequency f, e.g. read off signal generator/use c.r.o.
- Method to measure current in the coil, e.g. use ammeter.
- Method to measure voltage across the coil, e.g. use voltmeter/c.r.o.

Method of analysis (2 A-marks)

- Plot a graph of $1/I_0^2$ against f^2.
- $L = \sqrt{\dfrac{V_0^2 \times \text{gradient}}{4\pi^2}}$.

Safety considerations (1 S-mark)

- To prevent burns from hot coil, wear gloves/switch off when not in use/avoid touching.

Additional detail (4 D-marks)

- Measure R with ohmmeter.
- Keep R constant (e.g. by keeping temperature constant).
- Use low frequency a.c. to produce a large I_0.
- Produce a large I_0 to reduce percentage uncertainty in its measurement.
- Method to keep peak voltage V_0 across the coil constant, e.g. adjust signal generator/use of rheostat.
- Use a.c. ammeter and a.c. voltmeter.

  Note: In a.c. circuits, a.c. ammeter and voltmeter both give rms values of current and voltage, not peak values.

- Detail on changing rms value to peak value, i.e. for sinusoidal a.c.: peak value = rms value $\times \sqrt{2}$.
- y-intercept = R^2/V_0^2.
- Relationship is valid if graph is a straight line and y-intercept = R^2/V_0^2 (provided plotted graph is correct)
- Detail on determining period, and hence frequency, from c.r.o., i.e. period = length × time-base and frequency = 1/period.
- Detail on determining peak voltage V_0 from c.r.o., i.e. peak voltage = height × y-gain.

QUESTION 2: ANALYSIS, CONCLUSION AND EVALUATION

Generic Mark Scheme

(Before October/November 2015)

Breakdown of skills	**Mark allocation**
Approach to data analysis	1 mark
Table of results	2 marks
Graph	3 marks
Conclusion	4 marks
Treatment of uncertainties	5 marks

(After October/November 2015)

Breakdown of skills	**Minimum mark allocation***
Approach to data analysis	1 mark
Table of results	1 marks
Graph	2 marks
Conclusion	3 marks
Treatment of uncertainties	3 marks

*The remaining 5 marks are allocated across the skills in this grid and their allocation may vary from paper to paper.

In order to get an idea of the structure of question 2, first go through the complete questions given in the exercise section.

2.1 Determining Expressions of Gradient and Y-Intercept from a Linear Equation

If a *y* vs. *x* graph is drawn from an equation of the form:

$y = mx + c$

then:

gradient of the graph = m

y-intercept = c

Sample Question 2.1 *(P52/M/J/12)*

A student investigates how the resonant length L of a loaded wire varies with frequency f. It is suggested that f and L are related by the equation:

$$f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

where T is the tension in the wire and μ is a constant. A graph is plotted of f on the *y*-axis against $1/L$ on the *x*-axis. Determine an expression for the gradient in terms of T and μ. [1]

Solution

gradient $= \frac{1}{2}\sqrt{\frac{T}{\mu}}$

Note: In this section (i.e. question-2 section), instead of complete past-paper questions, only their respective components have been solved after explaining the required rules and methods. So, the sample question 2.1 solved above is not an example of complete question 2 that came in paper 52 of May/June 2012; rather, it is just a component of the actual question.

Sample Question 2.2 *(P53/O/N/11)*

A student is investigating how a mass m attached to a trolley by a string affects its velocity v. It is suggested that v and m are related by the equation:

$$mg = (m + M)\frac{v^2}{2h}$$

where M is the mass of the trolley, h is the height from which the mass m is released and g is the acceleration of free fall. A graph is plotted of v^2 on the *y*-axis against $\frac{m}{m+M}$ on the *x*-axis. Express the gradient in terms of g and h. [1]

Solution

gradient = $2gh$

Working

Sample Question 2.1

$$f = \frac{1}{2}\sqrt{\frac{T}{\mu}} \cdot \frac{1}{L}$$

Sample Question 2.2

$$v^2 = 2gh\left(\frac{m}{m+M}\right)$$

Sample Question 2.3 *(P53/O/N/12)*

A student investigates how the maximum speed *v* of a trolley varies with its total mass *M*. It is suggested that *v* and *M* are related by the equation:

$$v = A\sqrt{\frac{k}{M}}$$

where *A* is the length of the card attached to the trolley and *k* is the spring constant of the springs attached to it. A graph is plotted of v^2 on the *y*-axis against 1/*M* on the *x*-axis. Determine an expression for the gradient in terms of *A* and *k*. [1]

Solution

gradient = A^2k

Working

Sample Question 2.3

$$v^2 = A^2k \cdot \frac{1}{M}$$

Sample Question 2.4 *(P51/O/N/12)*

A student investigates how the minimum potential difference *V* required to cause an LED to emit light varies with its characteristic wavelength *λ*. It is suggested that *V* and *λ* are related by the equation:

$$\frac{hc}{\lambda} = B + eV$$

where *c* is the speed of light in a vacuum, *e* is the elementary charge, *h* is the Planck constant and *B* is a constant. A graph is plotted of *V* on the *y*-axis against 1/*λ* on the *x*-axis. Determine expressions for the gradient and *y*-intercept in terms of *B*, *c*, *e* and *h*. [1]

Solution

gradient = $\frac{hc}{e}$

y-intercept = $\frac{-B}{e}$

Sample Question 2.4

$$eV = \frac{hc}{\lambda} - B$$

$$\Rightarrow \quad V = \frac{hc}{e\lambda} - \frac{B}{e}$$

$$\Rightarrow \quad V = \frac{hc}{e} \cdot \frac{1}{\lambda} + \left(\frac{-B}{e}\right)$$

Sample Question 2.5 *(P51/M/J/14)*

A student investigates how the reading *V* on a voltmeter varies with the resistance *Q* of a resistor connected into a circuit. It is suggested that *V* and *Q* are related by the equation:

$$V = -ER\left(\frac{1}{P} + \frac{1}{Q}\right)$$

where *E* is the e.m.f. of the cell, and *P* and *R* are the resistances of other resistors connected into the circuit. A graph is plotted of *V*/*E* on the *y*-axis against 1/*Q* on the *x*-axis. Determine expressions for the gradient and the *y*-intercept in terms of *P* and *R*. [1]

Solution

gradient = – *R*

y-intercept = $\frac{-R}{P}$

Sample Question 2.5

$$\frac{V}{E} = -\frac{R}{Q} - \frac{R}{P}$$

$$\Rightarrow \quad \frac{V}{E} = (-R)\frac{1}{Q} + \left(\frac{-R}{P}\right)$$

2.2 Logarithmic Identities

1. $\lg (mn) = \lg m + \lg n$
2. $\lg\left(\frac{m}{n}\right) = \lg m - \lg n$
3. $\lg m^n = n \lg m$
4. $\lg 10 = 1$
5. $\ln e = 1$
   (where 'ln' is the symbol of natural logarithm, and $e = 2.718$)
6. $\ln e^x = x \ln e$
   $= x$
7. If:
   $\lg m = n$
   then:
   $m = 10^n$
8. If:
   $\ln m = n$
   then:
   $m = e^n$
9. If:
   $\lg m = n$
   then:
   $\lg\left(\frac{1}{m}\right) = -n$

Note: All above-stated identities hold true for natural logarithm as well.

Sample Question 2.6 *(P51/O/N/10)*

A student investigates how the period T of a simple pendulum depends on its length l. It is suggested that T and l are related by the equation:

$$T = al^b$$

where a and b are constants. A graph is plotted of $\lg T$ on the y-axis against $\lg l$ on the x-axis. Determine expressions for the gradient and y-intercept in terms of a and b. [1]

Solution

gradient = b

y-intercept = $\lg a$

Working

Sample Question 2.6

$\lg T = \lg a + \lg l^b$

$\Rightarrow \lg T = \lg a + b \lg l$

$\Rightarrow \lg T = b \lg l + \lg a$

Sample Question 2.7 *(P51/O/N/11)*

A scientist investigates how the period T of an orbit about the planet Jupiter varies with its radius r. It is suggested that T and r are related by the equation:

$$T^2 = kr^3$$

where k is a constant. A graph is plotted of lg T on the y-axis against lg r on the x-axis. Determine the value of the gradient and express the y-intercept in terms of k. [1]

Solution

gradient $= \frac{3}{2}$

y-intercept $= \frac{\lg k}{2}$

Working

Sample Question 2.7

$$\lg T^2 = \lg k + \lg r^3$$

$$\Rightarrow \quad 2 \lg T = \lg k + 3 \lg r$$

$$\Rightarrow \quad \lg T = \frac{\lg k}{2} + \frac{3}{2} \lg r$$

$$\Rightarrow \quad \lg T = \frac{3}{2} \lg r + \frac{\lg k}{2}$$

Sample Question 2.8 *(P05/M/J/08)*

A student investigates how the count rate R registered by a Geiger-Muller tube varies with the thickness x of a lead absorber placed between a radioactive source and the tube. It is suggested that R and x are related by the equation:

$$R = R_0 e^{-\rho \eta x}$$

where R_0 is the count rate with no absorbers, ρ is the density of lead and η is a constant. If a graph of ln R against x were plotted, what quantities in the above equation would the gradient and y-intercept represent? [1]

Solution

gradient $= -\rho\eta$

y-intercept $= \ln R_0$

Sample Question 2.8

$$\ln R = \ln R_0 + \ln e^{-\rho\eta x}$$

$$\Rightarrow \quad \ln R = \ln R_0 + (-\rho\eta x)$$

$$\Rightarrow \quad \ln R = (-\rho\eta)x + \ln R_0$$

2.3 Rules for Writing Column Heading

Examples

(a) The column heading for a quantity 'v^2', to be expressed in the units 'm^2/s^2', should be:

$$v^2 \,/\, m^2\, s^{-2}$$

(b) The column heading for a quantity $1/\lambda$, to be expressed in the units '$10^6\ m^{-1}$', should be:

$$(1/\lambda) \,/\, 10^6\ m^{-1}$$

(c) The column heading for 'lg v^2', where v^2 is to be expressed in the units 'm^2/s^2', should be:

$$\lg (v^2 \,/\, m^2\, s^{-2})$$

(d) The column heading for 'lg $(1/\lambda)$', where '$1/\lambda$' is to be expressed in the units '$10^6\ m^{-1}$', should be:

$$\lg [(1/\lambda) \,/\, 10^6\ m^{-1}]$$

Rules

1 In the column heading, the symbol of the quantity and its measuring unit should be separated by the distinguishing mark '/'.

2 Where logarithms are required, unit should be shown with the quantity whose logarithm is to be taken.

Note: The logarithm itself does not have a unit.

2.4 Rules for Determining the Number of Significant Figures (s.f.) and Decimal Places (d.p.)

Example

❖ If:

$r = 422 \times 10^6$ m

then:

no. of s.f. in r = 3

Rule 1

no. of s.f. in a quantity (r) = no. of digits in the number part of its value ($\underline{422} \times 10^6$ m)

Examples

(a) If:

$V = 1.80 \pm 0.05$ V

then:

no. of s.f. in V = 3

(b) If:

$T = 24 \pm 4$ s

then:

no. of s.f. in T = 2

(c) If:

$T = 1420 \pm 15$ s

then:

no. of s.f. in T = 3 (if it is a calculated quantity)

Rules

2 In a quantity, stated with its absolute uncertainty (also known as the actual uncertainty), last s.f. is the one that occupies the same decimal place (d.p.) as the first non-zero digit in its absolute uncertainty.

3 By definition of significant figures, the absolute uncertainty has only 1 s.f. (and it is always the first non-zero digit from the left). So, when stating the absolute uncertainty of a calculated quantity, it is always a good practice to round off it to the first non-zero digit (just to avoid confusion). For example, in the above example **(c)**, the absolute uncertainty in T could have been stated as ±20 s, instead of ±15 s. Although ±15 s is also acceptable, but ±20 s is just more appropriate.

Note: The absolute uncertainty is preferably stated to 1 s.f. only. The percentage uncertainty should, however, preferably be stated to 2 s.f., especially when the second digit is not zero (or does not remain as zero when the value is rounded off to the first two digits). For example, if the percentage uncertainty is calculated to be ±1.09%, then it should preferably be stated to 2 s.f. as ±1.1%.

Examples (of multiplication and division)

(a) If:

$\lambda = 950 \times 10^{-9}$ m

then:

$1/\lambda = 1.05263 \times 10^6 = 1.05 \times 10^6$ m^{-1}

(b) If:

$M = 1.25$ kg

then:

$1/M = 0.8 = 0.800$ kg^{-1}

(c) If:

$$d = 0.050 \text{ m}, t = 0.0581 \text{ s, and } v^2 = \frac{d^2}{t^2}$$

then:

$$v^2 = \frac{d^2}{t^2} = \frac{(0.050)^2}{(0.0581)^2} = 0.740607 = 0.74 \text{ m}^2 \text{ s}^{-2}$$

Rule 4

no. of s.f. retained in the calculated quantity = least no. of s.f. in the raw data

Note: This rule is a bit flexible. The number of s.f. to be retained in the calculated quantity may be either equal to or 'one more' than the least number of s.f. in the raw data, but it is usually preferable to keep the number of s.f. in the calculated quantity the same as the least number of s.f in the raw data. For example:

(i) if the calculated quantity may be stated to 3 or 4 s.f., then it is preferable to use 3 s.f.;

(ii) if the calculated quantity may be stated to 2 or 3 s.f., then it is preferable to use 2 s.f.;

(iii) **but** if the calculated quantity may be stated to 1 or 2 s.f., then it is preferable to use 2 s.f.

Examples (of addition and subtraction)

(a) If:

$x = 2.3$ m and $y = 1.946$ m

then:

$x + y = 2.3 + 1.946 = 4.246 = 4.2$ m

(b) If:

$x = 2.3$ m and $y = 1.956$ m

then:

$x - y = 2.3 - 1.956 = 0.344 = 0.3$ m

Rule 5

no. of d.p. retained in the calculated quantity = least no. of d.p. in the raw data

Example (of logarithm)

❖ If:

$x = 422 \times 10^6$ m

then:

$\lg x = \lg (422 \times 10^6) = 8.62531 = 8.625$

Rule 6

no. of d.p. in the calculated value of lg x = no. of s.f. in x

Note: This rule is also a bit flexible. The number of d.p. to be retained in the calculated value of lg x may be either equal to or 'one more' than the number of s.f. in x, but it is usually preferable to keep the number of d.p. in the calculated value of lg x the same as the number of s.f. in x.

Example (of antilogarithm)

❖ If:

$x = 4.15$ m

then:

$10^x = 10^{4.15} = 1.4125 \times 10^4 = 1.4 \times 10^4$

Rule 7

no. of s.f. in the calculated value of 10^x = no. of d.p. in x

Note: This rule is also a bit flexible. The number of s.f. to be retained in the calculated value of 10^x may be either equal to or 'one more' than the number of d.p. in x, but it is usually preferable to keep the number of s.f. in the calculated value of 10^x the same as the number of d.p. in x.

2.5 Rules for Rounding off the Calculated Quantities

Examples (of decimal-point values)

(a) 4.23189 m, when rounded off to the 2nd d.p., becomes 4.23 m

(b) 4.23589 m, when rounded off to the 2nd d.p., becomes 4.24 m

(c) 4.23689 m, when rounded off to the 2nd d.p., becomes 4.24 m

Rules

1 The last digit to be retained in the value remains unchanged if the very next digit (to its right) is less than 5.

2 The last digit to be retained in the value is increased by 1 if the very next digit (to its right) is 5 or greater than 5.

Examples (of whole-number values)

(a) 42318 m, when rounded off to 3 s.f., becomes 42300 m

(b) 42358 m, when rounded off to 3 s.f., becomes 42400 m

(c) 42368 m, when rounded off to 3 s.f., becomes 42400 m

Rules

3 The last digit to be retained in the value remains unchanged if the very next digit (to its right) is less than 5; whereas all the digits to be dropped are replaced with zeros.

4 The last digit to be retained in the value is increased by 1 if the very next digit (to its right) is 5 or greater than 5; whereas all the digits to be dropped are replaced with zeros.

Sample Question 2.9 *(P53/O/N/12)*

Values of M and t are given in Fig. 2.9.1.

M / kg	t / s	$(1/M)$ / kg^{-1}	v^2 / $m^2\ s^{-2}$
0.75	0.046 ± 0.002		
1.25	0.058 ± 0.002		
1.75	0.068 ± 0.002		
2.25	0.078 ± 0.002		
2.75	0.086 ± 0.002		
3.25	0.092 ± 0.002		

Figure 2.9.1

Calculate and record values of $(1/M)$ / kg^{-1} and v^2 / $m^2\ s^{-2}$ in Fig. 2.9.1.

(where: $v = \frac{d}{t}$, and $d = 5.0 \pm 0.1$ cm) [2]

Solution

M / kg	t / s	$(1/M)$ / kg^{-1}	v^2 / m^2 s^{-2}
0.75	0.046 ± 0.002	1.3	1.2
1.25	0.058 ± 0.002	0.800	0.74
1.75	0.068 ± 0.002	0.571	0.54
2.25	0.078 ± 0.002	0.444	0.41
2.75	0.086 ± 0.002	0.364	0.34
3.25	0.092 ± 0.002	0.308	0.30

Sample Question 2.10 *(P52/M/J/12)*

Values of f and L are given in Fig. 2.10.1.

f / Hz	L / 10^{-2} m	
256	54.5 ± 0.5	
294	48.0 ± 0.5	
330	42.5 ± 0.5	
350	40.0 ± 0.5	
396	35.5 ± 0.5	
440	32.0 ± 0.5	

Figure 2.10.1

Calculate and record values of $(1/L)$ / m^{-1} in Fig. 2.10.1. [2]

Solution

f / Hz	L / 10^{-2} m	$(1/L)$ / m^{-1}
256	54.5 ± 0.5	1.83
294	48.0 ± 0.5	2.08
330	42.5 ± 0.5	2.35
350	40.0 ± 0.5	2.50
396	35.5 ± 0.5	2.82
440	32.0 ± 0.5	3.13

Working

Sample Question 2.9

$$v^2 = \frac{d^2}{t^2}$$

$$= \frac{(0.050\text{ m})^2}{(0.046\text{ s})^2}$$

$$= 1.18\text{ m}^2\text{ s}^{-2}$$

$$= 1.2\text{ m}^2\text{ s}^{-2}$$

Sample Question 2.10

$$\frac{1}{L} = \frac{1}{54.5 \times 10^{-2}\text{ m}}$$

$$= 1.83486\text{ m}^{-1}$$

$$= 1.83\text{ m}^{-1}$$

Sample Question 2.11 *(P52/M/J/11)*

Values of R and V are given in Fig. 2.11.1.

R / Ω	V / V	$(1 / R) / 10^{-3}\ \Omega^{-1}$
150	14.4 ± 0.1	
220	10.4 ± 0.1	
330	7.4 ± 0.1	
470	5.6 ± 0.1	
680	4.4 ± 0.1	
860	3.8 ± 0.1	

Figure 2.11.1

Calculate and record values of $(1 / R) / 10^{-3}\ \Omega^{-1}$ in Fig. 2.11.1. [1]

Solution

R / Ω	V / V	$(1 / R) / 10^{-3}\ \Omega^{-1}$
150	14.4 ± 0.1	6.7
220	10.4 ± 0.1	4.5
330	7.4 ± 0.1	3.0
470	5.6 ± 0.1	2.1
680	4.4 ± 0.1	1.5
860	3.8 ± 0.1	1.2

Working

Sample Question 2.11

$$\frac{1}{R} = \frac{1}{150\ \Omega}$$
$$= 6.6666 \times 10^{-3}\ \Omega^{-1}$$
$$= 6.7 \times 10^{-3}\ \Omega^{-1}$$

Note: All values of R are whole-number values with '0' in the end; this implies that these zeros are not significant. So, all values of R have 2 significant figures, not 3.

Sample Question 2.12 *(P51/O/N/12)*

Values of λ and V are given in Fig. 2.12.1.

$\lambda / 10^{-9}$ m	V / V	
950	0.60 ± 0.05	
875	0.70 ± 0.05	
655	1.20 ± 0.05	
560	1.55 ± 0.05	
505	1.80 ± 0.05	
430	2.25 ± 0.05	

Figure 2.12.1

Calculate and record values of $(1/\lambda) / 10^{6}\ \text{m}^{-1}$ in Fig. 2.12.1. [2]

Sample Question 2.12

$$\frac{1}{\lambda} = \frac{1}{950 \times 10^{-9}\ \text{m}}$$
$$= 1.0526 \times 10^{6}\ \text{m}^{-1}$$
$$= 1.05 \times 10^{6}\ \text{m}^{-1}$$

Solution

λ / 10^{-9} m	V / V	$(1/\lambda)$ / 10^{6} m^{-1}
950	0.60 ± 0.05	1.05
875	0.70 ± 0.05	1.14
655	1.20 ± 0.05	1.53
560	1.55 ± 0.05	1.79
505	1.80 ± 0.05	1.98
430	2.25 ± 0.05	2.33

Sample Question 2.13 *(P51/O/N/10)*

Values of *l* and *t* are given in Fig. 2.13.1.

l / cm	*t* / s	*T* / s	lg (*l* / cm)	lg (*T* / s)
95.0	19.6 ± 0.2			
85.0	18.4 ± 0.2			
75.0	17.4 ± 0.2			
65.0	16.2 ± 0.2			
55.0	14.8 ± 0.2			
45.0	13.4 ± 0.2			

Figure 2.13.1

Calculate and record values of *T* / s, lg (*l* / cm) and lg (*T* / s) in Fig. 2.13.1.
(where: $T = \frac{t}{10}$) [2]

Solution

l / cm	*t* / s	*T* / s	lg (*l* / cm)	lg (*T* / s)
95.0	19.6 ± 0.2	1.96	1.978	0.292
85.0	18.4 ± 0.2	1.84	1.929	0.265
75.0	17.4 ± 0.2	1.74	1.875	0.241
65.0	16.2 ± 0.2	1.62	1.813	0.210
55.0	14.8 ± 0.2	1.48	1.740	0.170
45.0	13.4 ± 0.2	1.34	1.653	0.127

Sample Question 2.14 *(P51/O/N/11)*

Values of r and T are given in Fig. 2.14.1.

r / 10^6 m	T / 10^3 s	lg (r / m)	lg (T / s)
129	24 ± 4		
181	42 ± 4		
422	154 ± 8		
671	304 ± 8		
1070	590 ± 15		
1880	1420 ± 15		

Figure 2.14.1

Calculate and record values of lg (r / m) and lg (T / s) in Fig. 2.14.1. [2]

Solution

r / 10^6 m	T / 10^3 s	lg (r / m)	lg (T / s)
129	24 ± 4	8.111	4.38
181	42 ± 4	8.258	4.62
422	154 ± 8	8.625	5.188
671	304 ± 8	8.827	5.483
1070	590 ± 15	9.0294	5.771
1880	1420 ± 15	9.2742	6.152

Working

lg (r / m) = lg (129×10^6)

= 8.11058971

= 8.111

2.6 Rules for Determining Uncertainties

Examples (of determining absolute uncertainty)

(a) If:

$L = 40.0 \pm 0.5$ m

then:

$$\Delta\left(\frac{1}{L}\right) = \left(\frac{1}{L}\right)_{max} - \frac{1}{L} = \frac{1}{L_{min}} - \frac{1}{L} = \frac{1}{39.5} - \frac{1}{40.0} = 0.000316 = 0.0003\ \text{m}^{-1}$$

(b) If:

$d = 0.050 \pm 0.001$ m, $t = 0.086 \pm 0.002$ s, and $v^2 = \dfrac{d^2}{t^2}$

then:

$$\Delta v^2 = v^2_{max} - v^2 = \left(\frac{d^2}{t^2}\right)_{max} - \frac{d^2}{t^2} = \frac{d^2_{max}}{t^2_{min}} - \frac{d^2}{t^2} = \frac{(0.051)^2}{(0.084)^2} - \frac{(0.050)^2}{(0.086)^2} = 0.0306 = 0.03\ \text{m}^2\ \text{s}^{-2}$$

(c) If:

$R = 580 \pm 20\ \Omega$

then:

$$\Delta \ln R = (\ln R)_{max} - \ln R = \ln R_{max} - \ln R = \ln 600 - \ln 580 = 0.0339 = 0.03$$

(d) If:

$R = 330 \pm 20\ \Omega$

then:

$$\Delta \ln\left(\frac{1}{R}\right) = \ln\left(\frac{1}{R}\right)_{max} - \ln\left(\frac{1}{R}\right) = \ln\left(\frac{1}{R_{min}}\right) - \ln\left(\frac{1}{R}\right) = \ln\left(\frac{1}{310}\right) - \ln\left(\frac{1}{330}\right) = 0.0625 = 0.06$$

Rule 1

In any quantity, say x, the absolute uncertainty Δx may always be determined using the equation:

$$\Delta x = x_{max} - x$$

where x_{max} is the maximum possible value of the quantity x, and x is the most probable value.

Examples (of determining percentage uncertainty)

(a) If:

percentage uncertainty in $V = \frac{\Delta V}{V} \times 100 = 2\%$

percentage uncertainty in $I = \frac{\Delta I}{I} \times 100 = 3\%$

$P = VI$

then:

$$\frac{\Delta P}{P} \times 100 = \left(\frac{\Delta V}{V} \times 100\right) + \left(\frac{\Delta I}{I} \times 100\right) = 2 + 3 = 5\%$$

(b) If:

$$\frac{\Delta V}{V} \times 100 = 2\%, \ \frac{\Delta I}{I} \times 100 = 3\%, \text{ and } R = \frac{V}{I}$$

then:

$$\frac{\Delta R}{R} \times 100 = \left(\frac{\Delta V}{V} \times 100\right) + \left(\frac{\Delta I}{I} \times 100\right) = 2 + 3 = 5\%$$

(c) If:

$$\frac{\Delta m}{m} \times 100 = 2\%, \ \frac{\Delta v}{v} \times 100 = 3\%, \text{ and } E_k = \frac{1}{2}mv^2$$

then:

$$\frac{\Delta E_k}{E_k} \times 100 = \left(\frac{\Delta m}{m} \times 100\right) + 2\left(\frac{\Delta v}{v} \times 100\right) = 2 + 2(3) = 8\%$$

(d) If:

$$\frac{\Delta T}{T} \times 100 = 2\%, \ \frac{\Delta l}{l} \times 100 = 3\%, \text{ and } T = 2\pi\sqrt{\frac{l}{g}}$$

then:

$$g = \frac{4\pi^2 l}{T^2}$$

so:

$$\frac{\Delta g}{g} \times 100 = \left(\frac{\Delta l}{l} \times 100\right) + 2\left(\frac{\Delta T}{T} \times 100\right) = 3 + 2(2) = 7\%$$

(e) If:

$$\frac{\Delta l}{l} \times 100 = 2\%, \ \frac{\Delta g}{g} \times 100 = 3\%, \text{ and } T = 2\pi\sqrt{\frac{l}{g}}$$

then:

$$T = 2\pi\frac{l^{1/2}}{g^{1/2}}$$

so:

$$\frac{\Delta T}{T} \times 100 = \frac{1}{2}\left(\frac{\Delta l}{l} \times 100\right) + \frac{1}{2}\left(\frac{\Delta g}{g} \times 100\right) = \frac{1}{2}(2) + \frac{1}{2}(3) = 2.5\%$$

Rule 2

When two (or more) quantities are multiplied together (or divided one by the other) to give another quantity, the percentage uncertainty in the resultant quantity is determined by adding up the percentage uncertainties of the quantities multiplied (or divided). And, before the percentage uncertainties are added up, they are multiplied by the powers (indices) of their respective quantities.

Note: When determining the percentage uncertainty, always use absolute values (i.e. moduli) of the indices.

2.7 Rules for Expressing the Calculated Quantities with Uncertainties

Examples

(a) $V = 1.80 \pm 0.05$ V **appropriate**

(b) $V = 1.8 \pm 0.05$ V **inappropriate**

(c) $T = 1420 \pm 10$ s **appropriate**

(d) $T = 1421 \pm 10$ s **inappropriate**

(e) $L = 8.7 \pm 1.2$ s **appropriate**

Rule 1

In a quantity, to be stated with its absolute uncertainty, last s.f. should be the one that occupies the same decimal place (d.p.) as the first non-zero digit in its absolute uncertainty.

Note: By definition of significant figures, the absolute uncertainty has only 1 s.f.; so, it should preferably be rounded off to the first non-zero digit. But in example **(e)**, we used 2 non-zero digits to state the absolute uncertainty (±1.2 s) in L because: if the absolute uncertainty in L were rounded off to the first non-zero digit, the value of L would have to be stated to 1 s.f. only, and stating a quantity to 1 s.f. should preferably be avoided.

Sample Question 2.15 *(P52/M/J/12)*

Values of f, L and $1/L$ are given in Fig. 2.15.1.

f / Hz	L / 10^{-2} m	$(1/L)$ / m^{-1}
256	54.5 ± 0.5	1.83
294	48.0 ± 0.5	2.08
330	42.5 ± 0.5	2.35
350	40.0 ± 0.5	2.50
396	35.5 ± 0.5	2.82
440	32.0 ± 0.5	3.13

Figure 2.15.1

Calculate and record absolute uncertainties in $1/L$ in Fig. 2.15.1. [1]

Working

$$\Delta\left(\frac{1}{L}\right) = \left(\frac{1}{L}\right)_{max} - \frac{1}{L}$$

$$= \frac{1}{L_{min}} - \frac{1}{L}$$

$$= \frac{1}{54.0 \times 10^{-2}\ \text{m}} - \frac{1}{54.5 \times 10^{-2}\ \text{m}}$$

$$= 0.0169\ \text{m}^{-1}$$

$$= 0.02\ \text{m}^{-1}$$

Solution

f / Hz	L / 10^{-2} m	$(1/L)$ / m^{-1}
256	54.5 ± 0.5	1.83 ± 0.02
294	48.0 ± 0.5	2.08 ± 0.02
330	42.5 ± 0.5	2.35 ± 0.03
350	40.0 ± 0.5	2.50 ± 0.03
396	35.5 ± 0.5	2.82 ± 0.04
440	32.0 ± 0.5	3.13 ± 0.05

Sample Question 2.16 *(P52/M/J/11)*

Values of *R*, *V*, 1/*R* and *V*/*E* are given in Fig. 2.16.1.

R / Ω	V / V	$(1/R)$ / 10^{-3} Ω^{-1}	V/E
150	14.4 ± 0.1	6.7	9.0
220	10.4 ± 0.1	4.5	6.5
330	7.4 ± 0.1	3.0	4.6
470	5.6 ± 0.1	2.1	3.5
680	4.4 ± 0.1	1.5	2.8
860	3.8 ± 0.1	1.2	2.4

Figure 2.16.1

Calculate and record absolute uncertainties in *V*/*E* in Fig. 2.16.1.
(where: $E = 1.6 \pm 0.1$ V) [1]

Solution

R / Ω	V / V	$(1/R)$ / 10^{-3} Ω^{-1}	V/E
150	14.4 ± 0.1	6.7	9.0 ± 0.7
220	10.4 ± 0.1	4.5	6.5 ± 0.5
330	7.4 ± 0.1	3.0	4.6 ± 0.4
470	5.6 ± 0.1	2.1	3.5 ± 0.3
680	4.4 ± 0.1	1.5	2.8 ± 0.3
860	3.8 ± 0.1	1.2	2.4 ± 0.2

Working

Sample Question 2.16

$$\Delta\left(\frac{V}{E}\right) = \left(\frac{V}{E}\right)_{max} - \frac{V}{E}$$

$$= \frac{V_{max}}{E_{min}} - \frac{V}{E}$$

$$= \frac{14.5\text{ V}}{1.5\text{ V}} - \frac{14.4\text{ V}}{1.6\text{ V}}$$

$$= 0.6666$$

$$= 0.7$$

Sample Question 2.17 *(P53/O/N/12)*

Values of *M*, *t*, 1/*M* and v^2 are given in Fig. 2.17.1.

M / kg	t / s	$(1/M)$ / kg^{-1}	v^2 / m^2 s^{-2}
0.75	0.046 ± 0.002	1.3	1.2
1.25	0.058 ± 0.002	0.800	0.74
1.75	0.068 ± 0.002	0.571	0.54
2.25	0.078 ± 0.002	0.444	0.41
2.75	0.086 ± 0.002	0.364	0.34
3.25	0.092 ± 0.002	0.308	0.30

Figure 2.17.1

Calculate and record absolute uncertainties in v^2 in Fig. 2.17.1.
(where: $v = \frac{d}{t}$, and $d = 5.0 \pm 0.1$ cm) [1]

Solution

M / kg	t / s	$(1/M)$ / kg^{-1}	v^2 / m^2 s^{-2}
0.75	0.046 ± 0.002	1.3	1.2 ± 0.2
1.25	0.058 ± 0.002	0.800	0.74 ± 0.09
1.75	0.068 ± 0.002	0.571	0.54 ± 0.06
2.25	0.078 ± 0.002	0.444	0.41 ± 0.04
2.75	0.086 ± 0.002	0.364	0.34 ± 0.03
3.25	0.092 ± 0.002	0.308	0.30 ± 0.03

Working

Sample Question 2.17

$$\Delta v^2 = \Delta\left(\frac{d^2}{t^2}\right)$$

$$= \left(\frac{d^2}{t^2}\right)_{max} - \frac{d^2}{t^2}$$

$$= \frac{(0.051\,\text{m})^2}{(0.044\,\text{s})^2} - \frac{(0.050\,\text{m})^2}{(0.046\,\text{s})^2}$$

$$= 0.162\ \text{m}^2\,\text{s}^{-2}$$

$$= 0.2\ \text{m}^2\,\text{s}^{-2}$$

Sample Question 2.18 *(P51/O/N/14)*

Values of C, f, $1/C$ and f^2 are given in Fig. 2.18.1.

C / 10^{-4} F	f / Hz	$(1/C)$ / 10^3 F^{-1}	f^2 / 10^3 Hz^2
2.5 ± 10%	149	4.0	22.2
3.0 ± 10%	134	3.3	18.0
3.5 ± 10%	123	2.9	15.1
4.4 ± 10%	107	2.3	11.4
6.6 ± 10%	82	1.5	6.7
8.8 ± 10%	65	1.1	4.2

Figure 2.18.1

Calculate and record absolute uncertainties in $1/C$ in Fig. 2.18.1. [1]

Solution

C / 10^{-4} F	f / Hz	$(1/C)$ / 10^3 F^{-1}	f^2 / 10^3 Hz^2
2.5 ± 10%	149	4.0 ± 0.4	22.2
3.0 ± 10%	134	3.3 ± 0.3	18.0
3.5 ± 10%	123	2.9 ± 0.3	15.1
4.4 ± 10%	107	2.3 ± 0.2	11.4
6.6 ± 10%	82	1.5 ± 0.2	6.7
8.8 ± 10%	65	1.1 ± 0.1	4.2

Sample Question 2.18

$$\frac{\Delta(1/C)}{(1/C)} \times 100 = \frac{\Delta C}{C} \times 100$$

i.e.

$$\frac{\Delta(1/C)}{(1/C)} \times 100 = 10$$

$$\Rightarrow \quad \Delta(1/C) = \frac{(1/C)}{100} \times 10$$

$$\Rightarrow \quad \Delta(1/C) = \frac{(4.0 \times 10^3\ \text{F}^{-1})}{100} \times 10$$

$$= 0.40 \times 10^3\ \text{F}^{-1}$$

$$= 0.4 \times 10^3\ \text{F}^{-1}$$

Sample Question 2.19 *(P51/O/N/11)*

Values of r, T, lg (r / m) and lg (T / s) are given in Fig. 2.19.1.

r / 10^6 m	T / 10^3 s	lg (r / m)	lg (T / s)
129	24 ± 4	8.111	4.38
181	42 ± 4	8.258	4.62
422	154 ± 8	8.625	5.188
671	304 ± 8	8.827	5.483
1070	590 ± 15	9.0294	5.771
1880	1420 ± 15	9.2742	6.152

Figure 2.19.1

Calculate and record absolute uncertainties in lg (T / s) in Fig. 2.19.1. [1]

Solution

r / 10^6 m	T / 10^3 s	lg (r / m)	lg (T / s)
129	24 ± 4	8.111	4.38 ± 0.07
181	42 ± 4	8.258	4.62 ± 0.04
422	154 ± 8	8.625	5.188 ± 0.022
671	304 ± 8	8.827	5.483 ± 0.011
1070	590 ± 15	9.0294	5.771 ± 0.011
1880	1420 ± 15	9.2742	6.152 ± 0.005

Working

Sample Question 2.19

Δ lg (T / s) = [lg (T / s)]$_{max}$ − lg (T / s)

= lg (28×10^3) − lg (24×10^3)

= 0.0669

= 0.07

Sample Question 2.20 *(P51/O/N/10)*

Values of l, t, T, lg (l / cm) and lg (T / s) are given in Fig. 2.20.1.

l / cm	t / s	T / s	lg (l / cm)	lg (T / s)
95.0	19.6 ± 0.2	1.96	1.978	0.292
85.0	18.4 ± 0.2	1.84	1.929	0.265
75.0	17.4 ± 0.2	1.74	1.875	0.241
65.0	16.2 ± 0.2	1.62	1.813	0.210
55.0	14.8 ± 0.2	1.48	1.740	0.170
45.0	13.4 ± 0.2	1.34	1.653	0.127

Figure 2.20.1

Calculate and record absolute uncertainties in lg (T / s) in Fig. 2.20.1.
(where: $T = \frac{t}{10}$) [1]

Solution

l / cm	t / s	T / s	lg (l / cm)	lg (T / s)
95.0	19.6 ± 0.2	1.96	1.978	0.292 ± 0.004
85.0	18.4 ± 0.2	1.84	1.929	0.265 ± 0.005
75.0	17.4 ± 0.2	1.74	1.875	0.241 ± 0.005
65.0	16.2 ± 0.2	1.62	1.813	0.210 ± 0.005
55.0	14.8 ± 0.2	1.48	1.740	0.170 ± 0.006
45.0	13.4 ± 0.2	1.34	1.653	0.127 ± 0.006

Working

Sample Question 2.20

$\Delta \lg T = (\lg T)_{max} - \lg T$

$= [\lg (t/10)]_{max} - \lg (t/10)$

$= \lg (19.8/10) - \lg (19.6/10)$

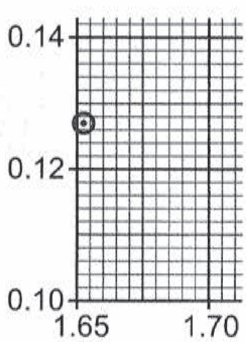
$= 0.0044$

$= 0.004$

2.8 Rules for Plotting the Points, and Drawing the Error-Bars

1 **All** points should be plotted.

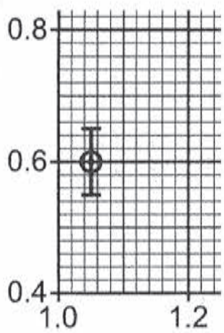
2 The plotting should be accurate to within half a small square. For example, the point (1.653, 0.127) should be plotted as shown below:



Note: Avoid blobbing; it is penalised if the size of blob is equal to or greater than half the size of a small square.

3 Error bars should be drawn for **all** plotted points.

4 The drawing of error bars should also be accurate to within half a small square. For example, the error bar for the point (1.05, 0.60 ± 0.05) should be drawn as shown below:



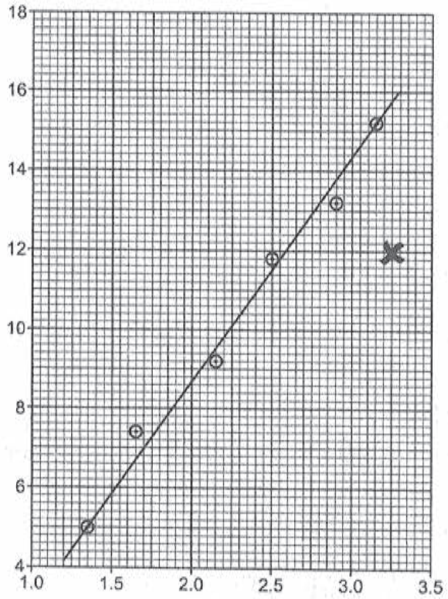
$$\text{error bar} = \frac{2 \times \text{uncertainty}}{\text{size of 1 small square}}$$

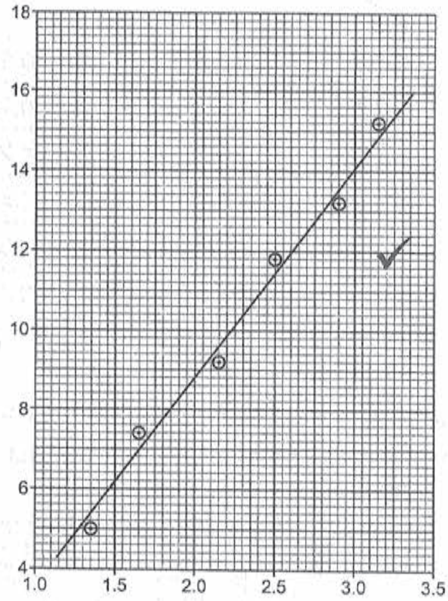
$$= \frac{2 \times 0.05}{0.02}$$

$= 5$ small squares

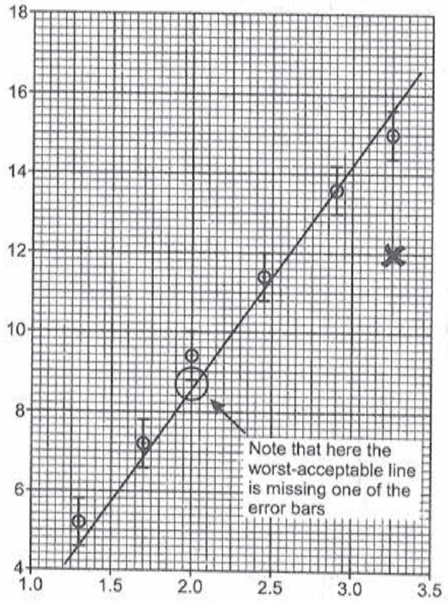
2.9 Rules for Drawing the Line of Best Fit and the Worst Acceptable Line

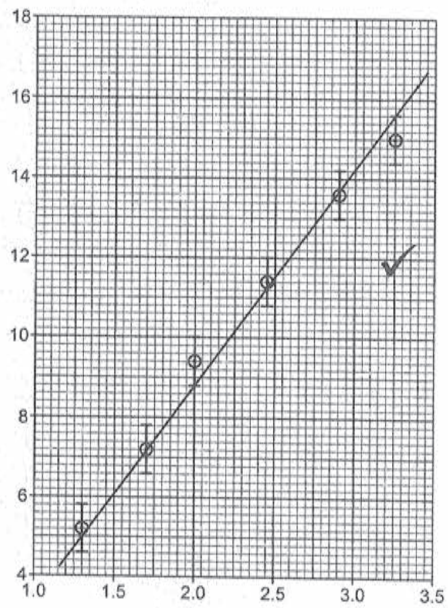
1 The line of best fit should either pass through (almost) all the plotted points, or have an **even** distribution of the plotted points on both sides with the overall **minimum** possible mean deviation, as shown below:





2 The worst acceptable line should be either the steepest possible line or the shallowest possible line passing through **all** error bars. Candidates may draw either the steepest or the shallowest worst acceptable line, but it is usually preferable to draw the steepest worst acceptable line. It should be noted that the worst acceptable line does not have to pass through the top of the top error bar and bottom of the bottom error bar, or bottom of the top error bar and top of the bottom error bar, as shown below:





Notes

(i) The line of best fit and the worst acceptable line should both be clearly labelled.

(ii) There should be no bent (kink) in the lines; so candidates should use a transparent 30 cm rule when drawing the lines.

Sample Question 2.21 *(P51/O/N/12)*

Values of λ, V and $1/\lambda$ are given in Fig. 2.21.1.

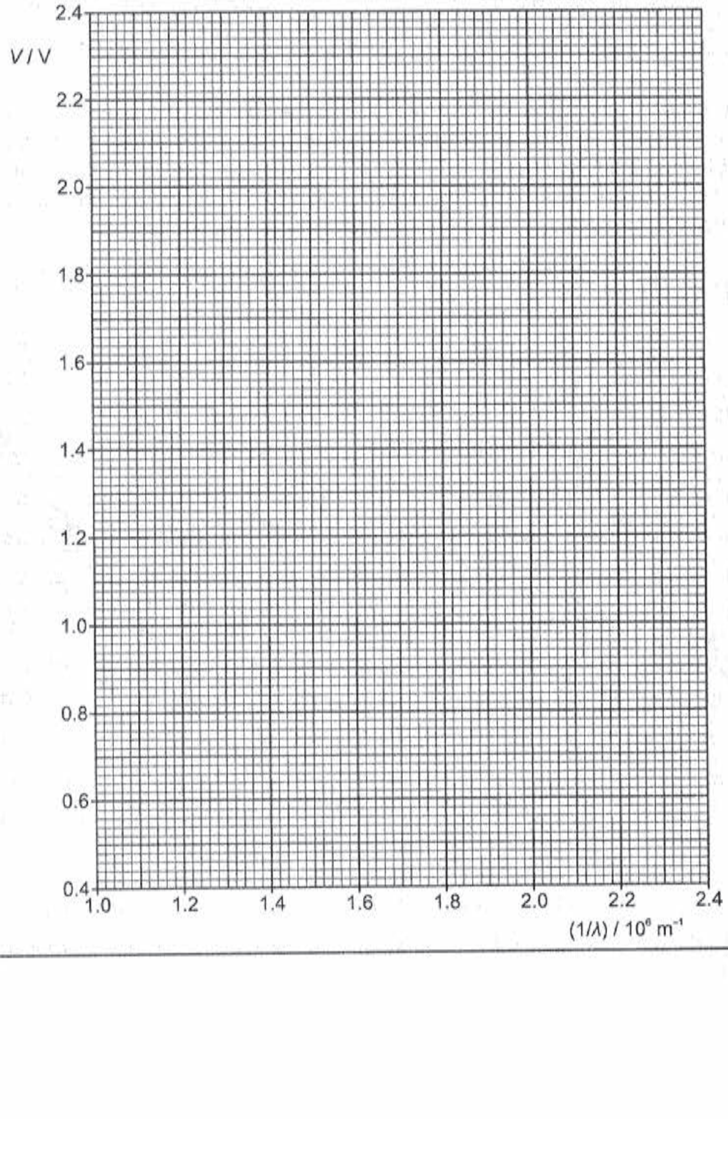
λ / 10^{-9} m	V / V	$(1/\lambda)$ / 10^{6} m^{-1}
950	0.60 ± 0.05	1.05
875	0.70 ± 0.05	1.14
655	1.20 ± 0.05	1.53
560	1.55 ± 0.05	1.79
505	1.80 ± 0.05	1.98
430	2.25 ± 0.05	2.33

Figure 2.21.1

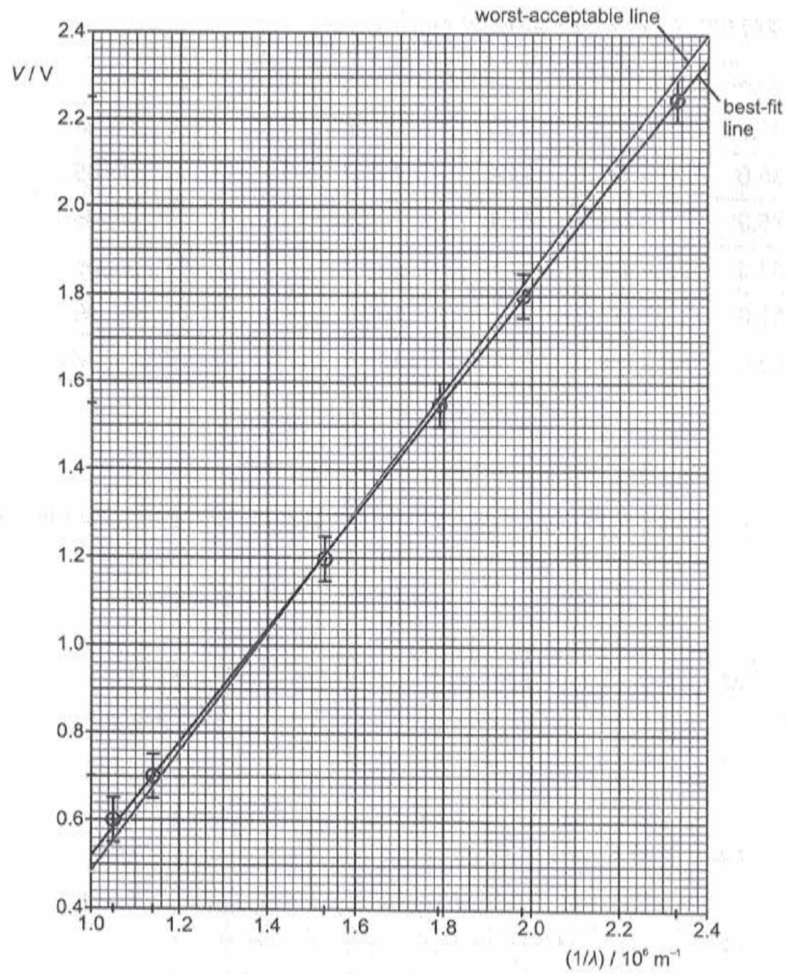
Use Fig. 2.21.1 to:

(i) plot a graph of V / V against $(1/\lambda)$ / 10^6 m^{-1}. Include error bars for V. [2]

(ii) Draw the straight line of best fit and a worst acceptable straight line on your graph. Both lines should be clearly labelled. [2]



Solution


worst-acceptable line
best-fit line
V / V
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
(1/λ) / 10⁶ m⁻¹

Sample Question 2.22 *(P51/O/N/10)*

Values of l, t, T, lg (l / cm) and lg (T / s) are given in Fig. 2.22.1.

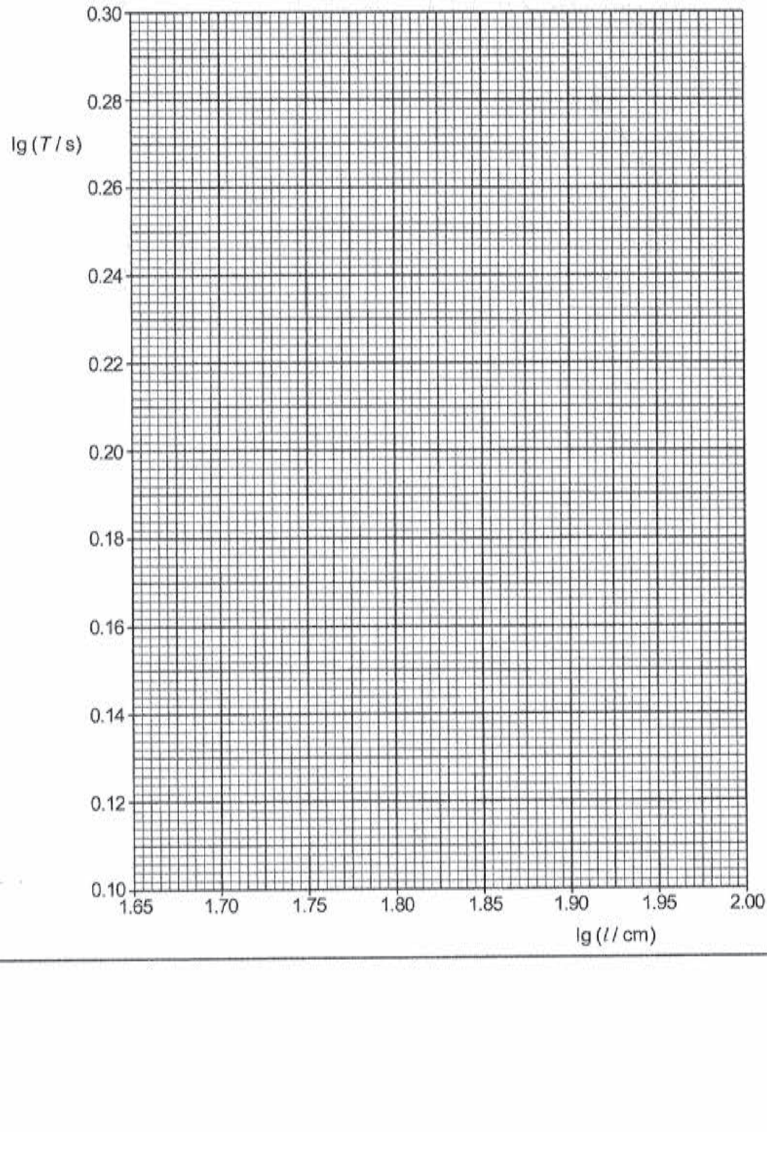
l / cm	t / s	T / s	lg (l / cm)	lg (T / s)
95.0	19.6 ± 0.2	1.96	1.978	0.292 ± 0.004
85.0	18.4 ± 0.2	1.84	1.929	0.265 ± 0.005
75.0	17.4 ± 0.2	1.74	1.875	0.241 ± 0.005
65.0	16.2 ± 0.2	1.62	1.813	0.210 ± 0.005
55.0	14.8 ± 0.2	1.48	1.740	0.170 ± 0.006
45.0	13.4 ± 0.2	1.34	1.653	0.127 ± 0.006

Figure 2.22.1

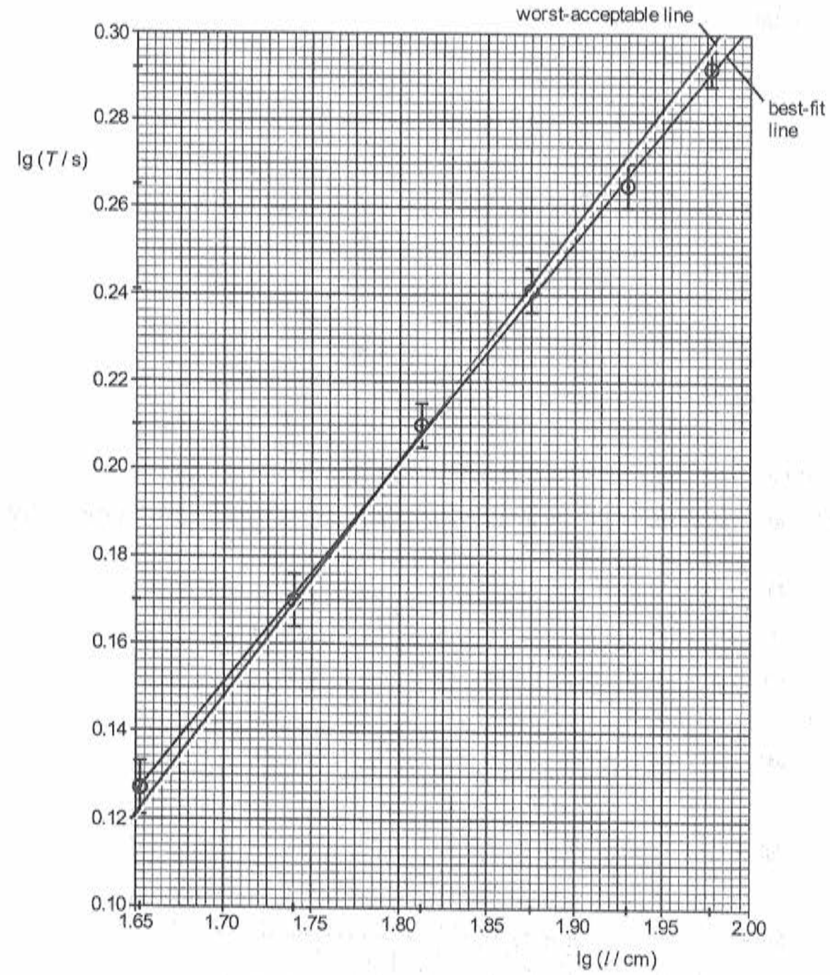
Use Fig. 2.22.1 to:

(i) plot a graph of lg (T / s) against lg (l / cm). Include error bars for lg (T / s). [2]

(ii) Draw the straight line of best fit and a worst acceptable straight line on your graph. Both lines should be clearly labelled. [2]



Solution


worst-acceptable line
best-fit line
lg (T / s)
0.30
0.28
0.26
0.24
0.22
0.20
0.18
0.16
0.14
0.12
0.10
1.65
1.70
1.75
1.80
1.85
1.90
1.95
2.00
lg (l / cm)

Sample Question 2.23 *(P52/M/J/12)*

Values of f, L and $1/L$ are given in Fig. 2.23.1.

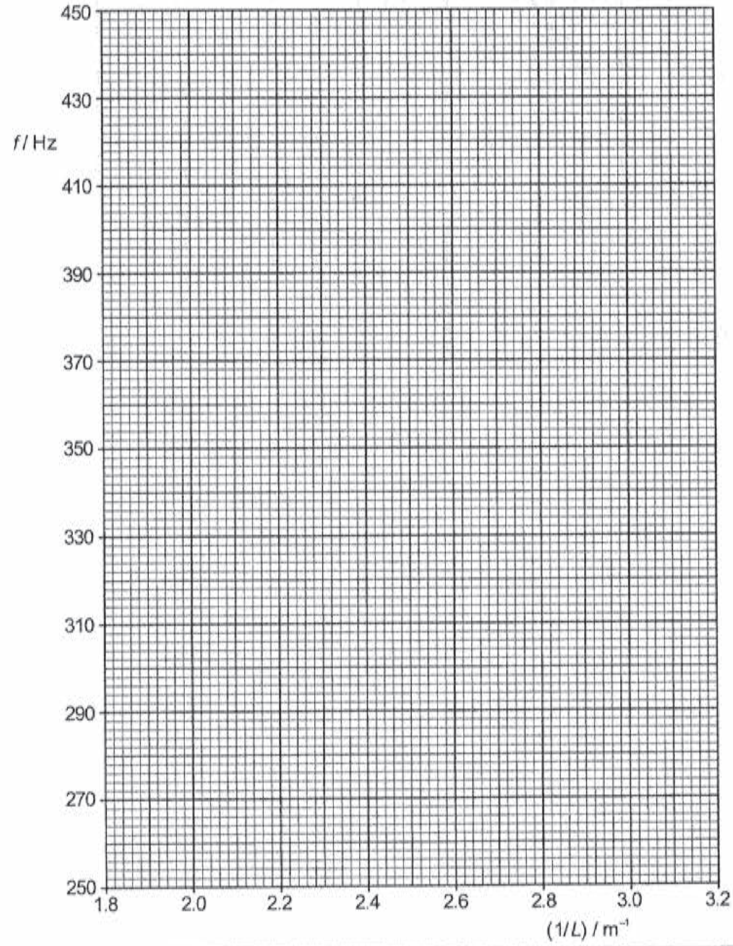
f / Hz	L / 10^{-2} m	$(1/L)$ / m^{-1}
256	54.5 ± 0.5	1.83 ± 0.02
294	48.0 ± 0.5	2.08 ± 0.02
330	42.5 ± 0.5	2.35 ± 0.03
350	40.0 ± 0.5	2.50 ± 0.03
396	35.5 ± 0.5	2.82 ± 0.04
440	32.0 ± 0.5	3.13 ± 0.05

Figure 2.23.1

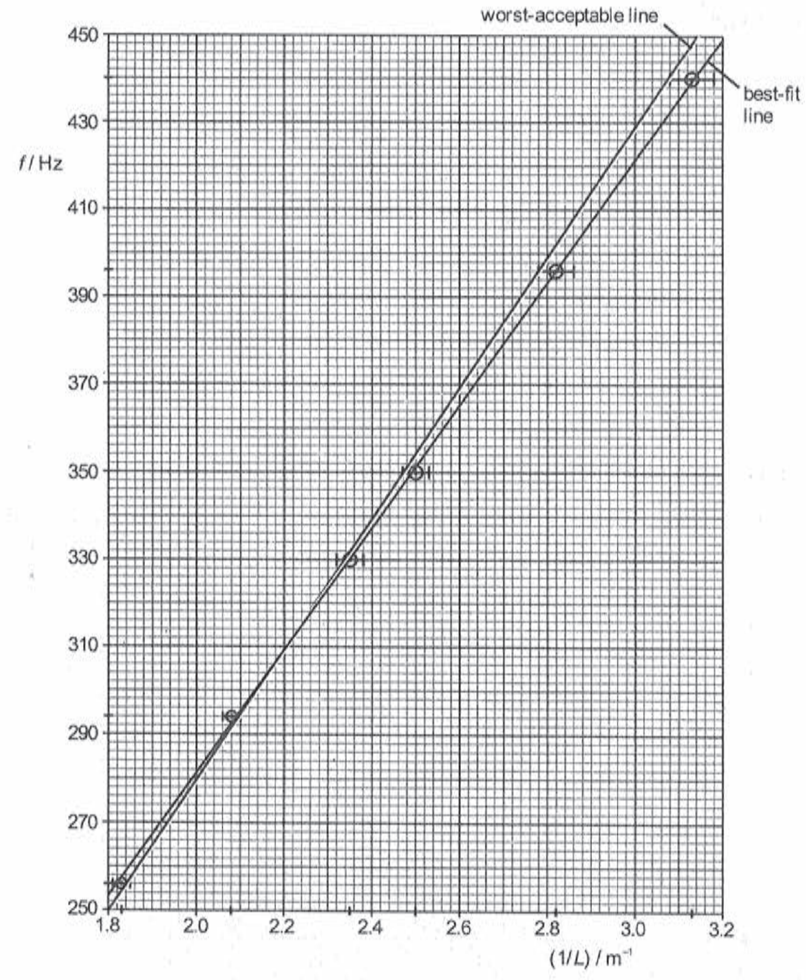
Use Fig. 2.23.1 to:

(i) plot a graph of f / Hz against $(1/L)$ / m^{-1}. Include error bars for $1/L$. [2]

(ii) Draw the straight line of best fit and a worst acceptable straight line on your graph. Both lines should be clearly labelled. [2]



Solution


worst-acceptable line
best-fit line
f / Hz
450
430
410
390
370
350
330
310
290
270
250
1.8
2.0
2.2
2.4
2.6
2.8
3.0
3.2
(1/L) / m⁻¹

Sample Question 2.24 *(P05/M/J/08)*

Values of x, R and $\ln R$ are given in Fig. 2.24.1.

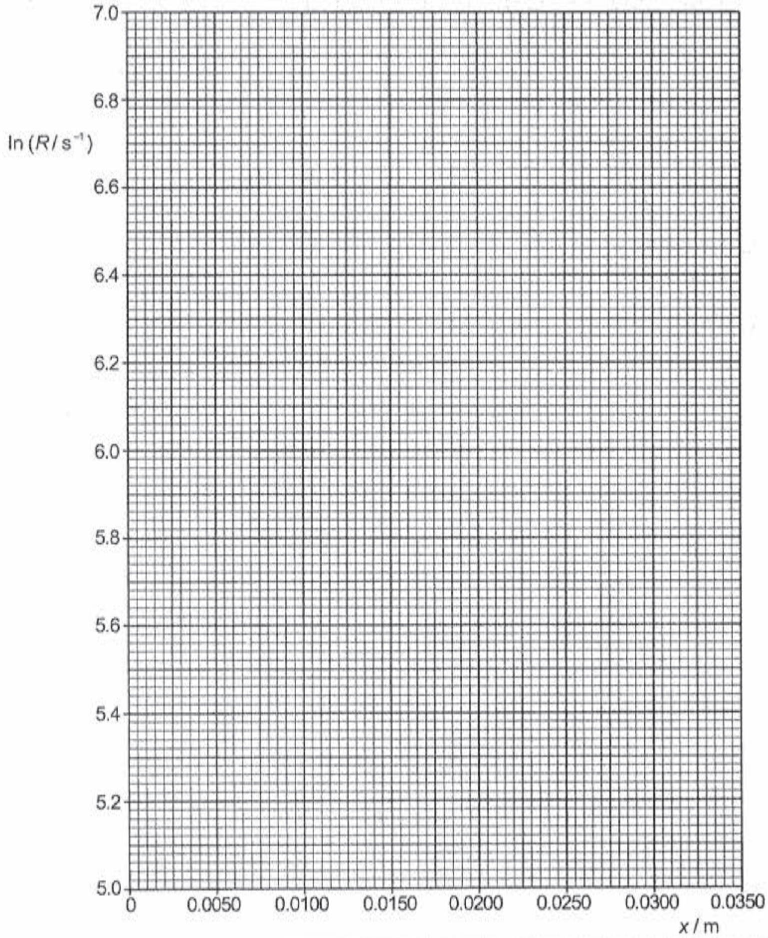
x / m	R / s^{-1}	$\ln (R / s^{-1})$
0.0050	750 ± 20	6.62 ± 0.03
0.0100	580 ± 20	6.36 ± 0.03
0.0150	430 ± 20	6.06 ± 0.05
0.0200	330 ± 20	5.80 ± 0.06
0.0250	250 ± 20	5.52 ± 0.08
0.0300	190 ± 20	5.25 ± 0.10

Figure 2.24.1

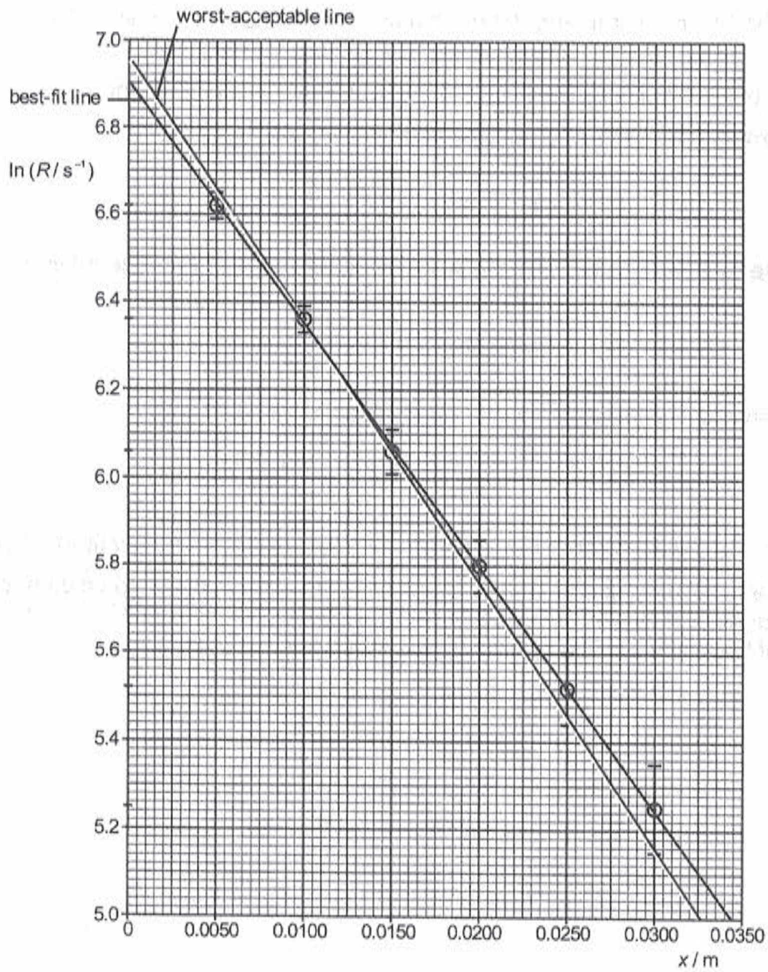
Use Fig. 2.24.1 to:

(i) plot a graph of $\ln R$ against x. Include error bars for $\ln R$. [2]

(ii) Draw the straight line of best fit and a worst acceptable straight line on your graph. Both lines should be clearly labelled. [2]



Solution


worst-acceptable line
best-fit line
ln (R / s⁻¹)
7.0
6.8
6.6
6.4
6.2
6.0
5.8
5.6
5.4
5.2
5.0
0
0.0050
0.0100
0.0150
0.0200
0.0250
0.0300
0.0350
x / m

2.10 Determining Gradient and Uncertainty in the Gradient from the Graph

1 Take two points on the line of best fit and draw a triangle which should be at least half the length of the line of best fit drawn.

2 Note the coordinates of the two points taken on the line of best fit: x_{1b}, y_{1b}, x_{2b} and y_{2b}.

3 Now determine the gradient of the line of best fit using the formula:

$$(\text{gradient})_{\text{best}} = \frac{y_{2b} - y_{1b}}{x_{2b} - x_{1b}}$$

4 Repeat the procedure from step **1** to **3** for the worst acceptable line, and determine its gradient using the formula:

$$(\text{gradient})_{\text{worst}} = \frac{y_{2w} - y_{1w}}{x_{2w} - x_{1w}}$$

5 Now determine the (absolute) uncertainty in the gradient using the equation:

$$\Delta(\text{gradient}) = (\text{gradient})_{\text{worst}} - (\text{gradient})_{\text{best}}$$

Notes:

(i) In this section, marks are usually awarded for the right method only, not for the values calculated.

(ii) Although no mark is allocated for stating an appropriate unit after the calculated value of gradient here, yet it might prove beneficial (for the subsequent components) to calculate the unit of the gradient from the graph and state the value of the gradient along with the appropriate unit.

2.11 Determining *Y*-intercept and Uncertainty in the *Y*-intercept from the Graph

1 Determine the *y*-intercept of the line of best fit using the equation:

$$(y\text{-intercept})_{best} = y_{2b} - (\text{gradient})_{best}\, x_{2b}$$

where x_{2b} and y_{2b} are the coordinates of one of the two points taken on the line of best fit to determine its gradient.

2 Likewise determine the *y*-intercept of the worst acceptable line using the equation:

$$(y\text{-intercept})_{worst} = y_{2w} - (\text{gradient})_{worst}\, x_{2w}$$

3 Now determine the (absolute) uncertainty in *y*-intercept using the equation:

$$\Delta(y\text{-intercept}) = (y\text{-intercept})_{best} - (y\text{-intercept})_{worst}$$

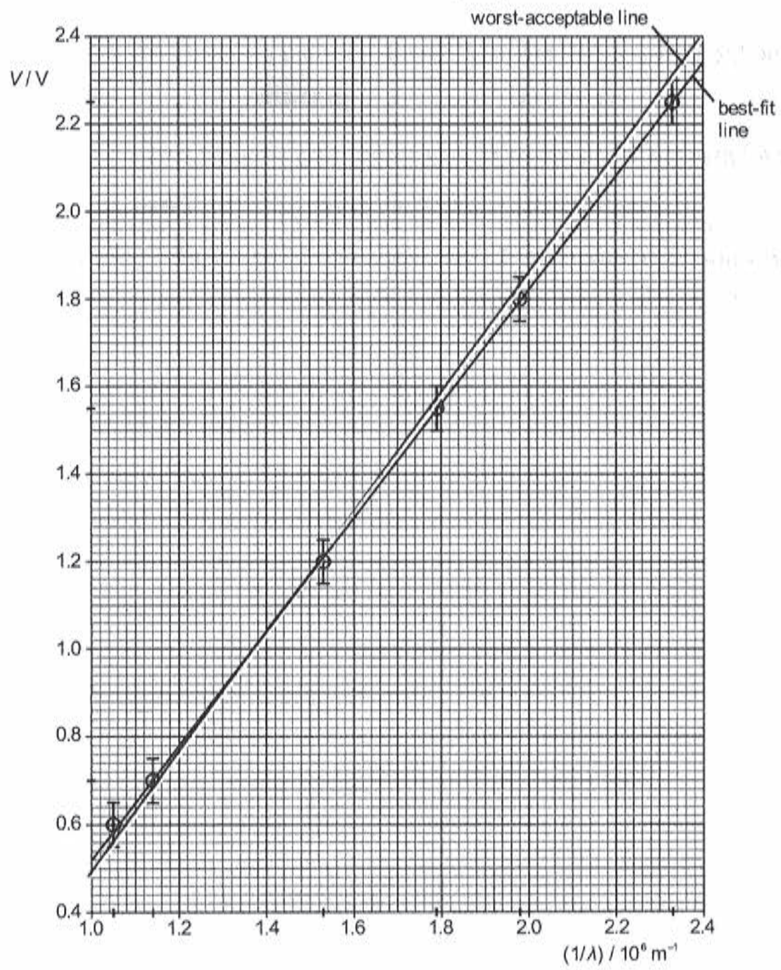
Note: Here again, marks are usually awarded for the right method only, not for the values calculated.

Sample Question 2.25 *(P51/O/N/12)*

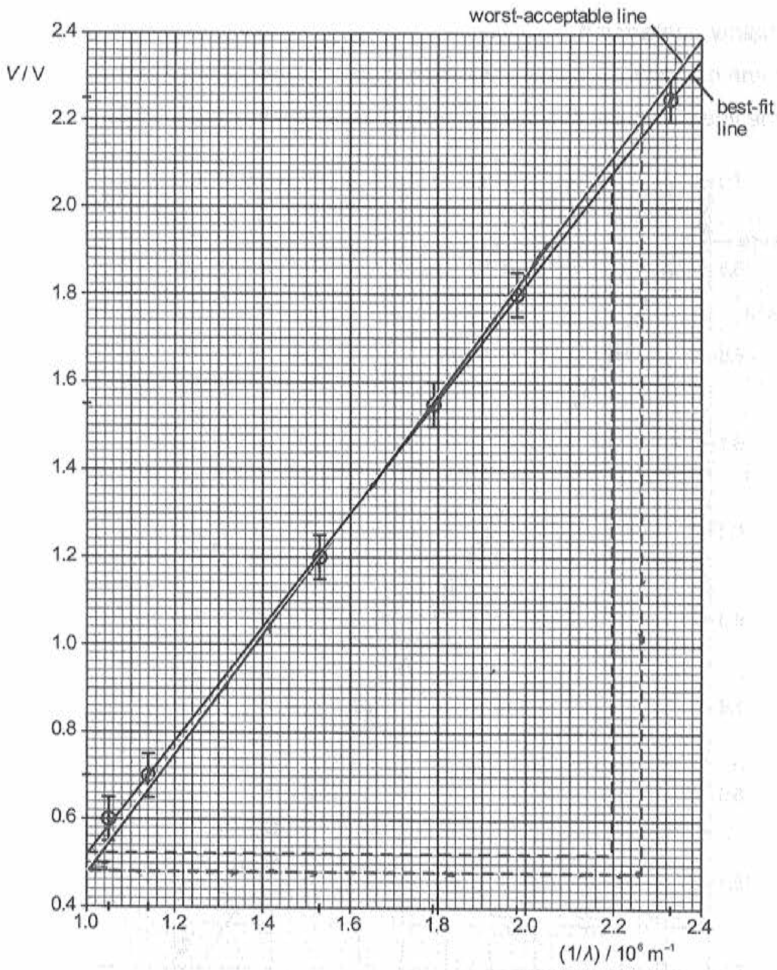
From the graph (given below), determine:

(i) the gradient of the line of best fit. Include the uncertainty in your answer. [2]

(ii) the *y*-intercept of the line of best fit. Include the uncertainty in your answer. [2]



Solution



(i) $(\text{gradient})_{best} = \left(\dfrac{2.08 - 0.52}{2.20 \times 10^6 - 1.00 \times 10^6}\right) \text{V m} = 1.30 \times 10^{-6}\ \text{V m}$

$(\text{gradient})_{worst} = \left(\dfrac{2.20 - 0.48}{2.26 \times 10^6 - 1.00 \times 10^6}\right) \text{V m} = 1.37 \times 10^{-6}\ \text{V m}$

$\Delta(\text{gradient}) = (1.37 \times 10^{-6}) - (1.30 \times 10^{-6}) = 0.07 \times 10^{-6}\ \text{V m}$

So:

$\text{gradient} = (1.30 \pm 0.07) \times 10^{-6}\ \text{V m}$

(ii) $(y\text{-intercept})_{best} = 2.08 - (1.30 \times 10^{-6})(2.20 \times 10^6) = -0.78\ \text{V}$

$(y\text{-intercept})_{worst} = 2.20 - (1.37 \times 10^{-6})(2.26 \times 10^6) = -0.896\ \text{V}$

$\Delta(y\text{-intercept}) = (-0.78) - (-0.896) = 0.116\ \text{V}$

So:

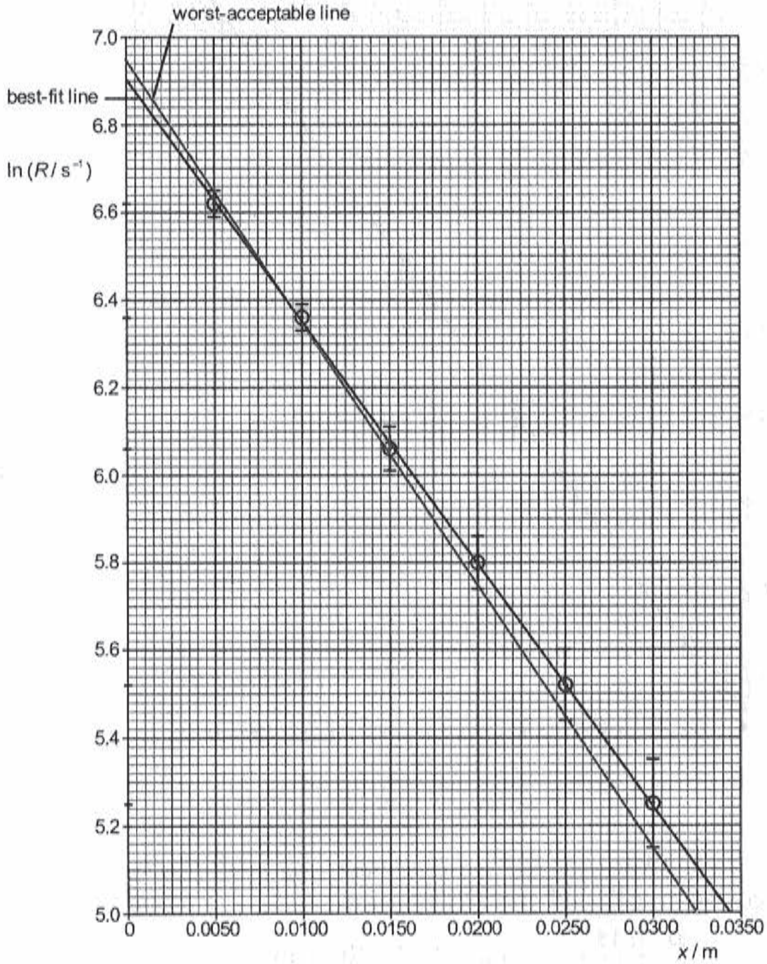
$y\text{-intercept} = -0.78 \pm 0.12\ \text{V}$

Sample Question 2.26 *(P05/M/J/08)*

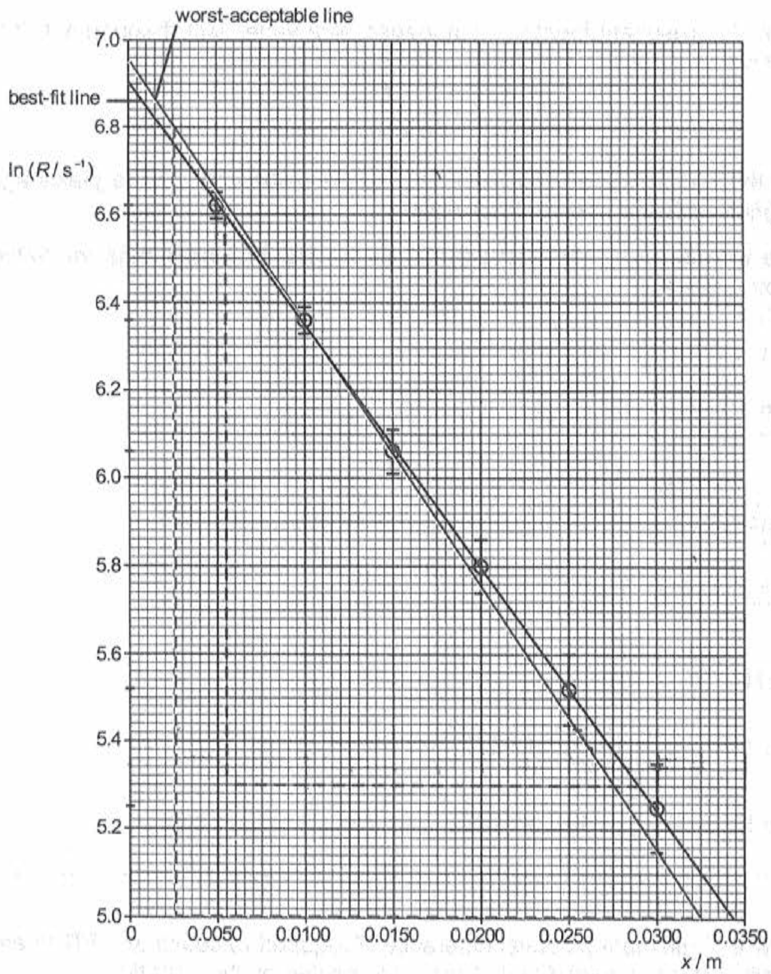
From the graph (given below), determine:

(i) the gradient of the line of best fit. Include the error in your answer. [2]

(ii) the *y*-intercept of the line of best fit. Include the error in your answer. [2]



Solution



(i) $(\text{gradient})_{\text{best}} = \left(\dfrac{5.30 - 6.60}{0.0290 - 0.0055}\right) \text{m}^{-1} = -55.3\ \text{m}^{-1}$

$(\text{gradient})_{\text{worst}} = \left(\dfrac{5.00 - 6.80}{0.0325 - 0.0025}\right) \text{m}^{-1} = -60.0\ \text{m}^{-1}$

$\Delta(\text{gradient}) = (-55.3) - (-60.0) = 4.7\ \text{m}^{-1}$

So:

gradient $= -55 \pm 5\ \text{m}^{-1}$

(ii) $(y\text{-intercept})_{\text{best}} = 6.90$

$(y\text{-intercept})_{\text{worst}} = 6.95$

$\Delta(y\text{-intercept}) = 6.95 - 6.90 = 0.05$

So:

y-intercept $= 6.90 \pm 0.05$

Sample Question 2.27 *(P52/M/J/12)*

A student investigates how the resonant length L of a loaded wire varies with frequency f. It is suggested that f and L are related by the equation:

$$f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

where T is the tension in the wire and μ is a constant. A graph is plotted of f on the y-axis against $1/L$ on the x-axis. The gradient of the graph is found to be 142 ± 7 Hz m.

(i) The tension T in the wire is 30 ± 3 N. Using the value of gradient, determine the value of μ. Include an appropriate unit in your answer.

(where: gradient $= \frac{1}{2}\sqrt{\frac{T}{\mu}}$) [2]

(ii) Determine the percentage uncertainty in μ. [1]

Solution

(i) $$\mu = \frac{T}{4 \times (\text{gradient})^2} = \frac{30}{4 \times (142)^2} = 3.7 \times 10^{-4}$$

$$\text{unit of } \mu = \frac{\text{N}}{(\text{Hz m})^2} = \text{N Hz}^{-2}\,\text{m}^{-2}$$

So:

$$\mu = 3.7 \times 10^{-4} \text{ N Hz}^{-2}\,\text{m}^{-2}$$

(ii) $$\frac{\Delta\mu}{\mu} \times 100 = \left(\frac{\Delta T}{T} \times 100\right) + 2\left(\frac{\Delta(\text{gradient})}{(\text{gradient})} \times 100\right)$$

$$= \left(\frac{3}{30} \times 100\right) + 2\left(\frac{7}{142} \times 100\right) = 20\%$$

Sample Question 2.28 *(P51/O/N/12)*

A student investigates how the minimum potential difference V required to cause an LED to emit light varies with its characteristic wavelength λ. It is suggested that V and λ are related by the equation:

$$V = \frac{hc}{e\lambda} - \frac{B}{e}$$

where c is the speed of light in a vacuum, e is the elementary charge, h is the Planck constant and B is a constant. A graph is plotted of V on the y-axis against $1/\lambda$ on the x-axis. The y-intercept of the graph is found to be -0.78 ± 0.11 V. Using the value of y-intercept, determine a value for B. Include an appropriate unit and the absolute uncertainty in your answer.

(where: y-intercept $= \frac{-B}{e}$, and $e = 1.6 \times 10^{-19}$ C) [2]

Solution

$B = -e \times (y\text{-intercept}) = -(1.6 \times 10^{-19}) \times (-0.78) = 1.2 \times 10^{-19}$

$B_{max} = -(1.6 \times 10^{-19}) \times (-0.89) = 1.4 \times 10^{-19}$

$\Delta B = (1.4 \times 10^{-19}) - (1.2 \times 10^{-19}) = 0.2 \times 10^{-19}$

So:

$B = (1.2 \pm 0.2) \times 10^{-19}$ C V

Sample Question 2.29 *(P05/M/J/08)*

A student investigates how the count rate R registered by a Geiger-Muller tube varies with the thickness x of a lead absorber placed between a radioactive source and the tube. It is suggested that R and x are related by the equation:

$$R = R_0 e^{-\rho\eta x}$$

where R_0 is the count rate with no absorbers, ρ is the density of lead and η is a constant. A graph is plotted of $\ln R$ on the y-axis against x on the x-axis. The gradient of the graph is found to be $-55 \pm 5\ \text{m}^{-1}$.

(a) The density of lead is given as 11300 kg m^{-3}. Using the value of gradient, determine the value of η. Include an appropriate unit and the error in your answer.
(where: gradient = $-\rho\eta$) [3]

(b) Use your answer from **(a)** to determine the thickness of lead, in m, required to reduce R to 10% of R_0. Include the error in your value. [2]

Working

$$\text{unit of } \eta = \frac{\text{m}^{-1}}{\text{kg m}^{-3}} = \text{kg}^{-1}\ \text{m}^2$$

Solution

(a)
$$\eta = \frac{-(\text{gradient})}{\rho} = \frac{-(-55)}{11300} = 4.9 \times 10^{-3}$$

$$\eta_{max} = \frac{-(-60)}{11300} = 5.3 \times 10^{-3}$$

$$\Delta\eta = (5.3 \times 10^{-3}) - (4.9 \times 10^{-3}) = 0.4 \times 10^{-3}$$

So:

$$\eta = (4.9 \pm 0.4) \times 10^{-3}\ \text{kg}^{-1}\ \text{m}^2$$

(b)
$$R = 10\% \text{ of } R_0 = 0.1R_0$$

$$\because \quad R = R_0 e^{-\rho\eta x}$$

$$\therefore \quad 0.1R_0 = R_0 e^{-\rho\eta x}$$

$$\Rightarrow \quad x = \frac{-\ln 0.1}{\rho\eta} = \frac{-\ln 0.1}{(11300)(4.9 \times 10^{-3})} = 0.042$$

$$x_{max} = \frac{-\ln 0.1}{(11300)(4.5 \times 10^{-3})} = 0.045$$

$$\Delta x = 0.045 - 0.042 = 0.003$$

So:

$$x = 0.042 \pm 0.003\ \text{m}$$

Sample Question 2.30 *(P52/M/J/11)*

A student investigates how the reading *V* on a voltmeter varies with resistance *R* of a resistor connected into a circuit. It is suggested that *V* and *R* are related by the equation:

$$\frac{V}{E} = \frac{F}{R} + 1$$

where *F* is the resistance of another fixed resistor in the circuit and *E* is the e.m.f. of the cell supplying power to the circuit. A graph is plotted of *V/E* on the *y*-axis against 1/*R* on the *x*-axis. The gradient of the graph is found to be 1200 ± 200 Ω. For one measurement, *R* has a value of 120 Ω ± 5%. Using the relationship given, determine the value of *V/E*. Include the absolute uncertainty in your answer.
(where: gradient = *F*) [2]

Solution

$F = 1200 \pm 200\ \Omega$

$R = 120\ \Omega \pm 5\%$

$$= 120 \pm \frac{5}{100} \times 120$$

$$= 120 \pm 6\ \Omega$$

$$\frac{V}{E} = \frac{F}{R} + 1 = \frac{1200}{120} + 1 = 11$$

$$\left(\frac{V}{E}\right)_{max} = \frac{1400}{114} + 1 = 13$$

$$\Delta\left(\frac{V}{E}\right) = 13 - 11 = 2$$

So:

$$\frac{V}{E} = 11 \pm 2$$

Sample Question 2.31 *(P51/O/N/10)*

A student investigates how the period T of a simple pendulum depends on its length l. It is suggested that T and l are related by the equation:

$$\lg T = b \lg l + \lg a$$

where a and b are constants. A graph is plotted of $\lg T$ on the y-axis against $\lg l$ on the x-axis. The gradient of the graph is found to be 0.50 ± 0.03, and the y-intercept, -0.70 ± 0.05. Using the values of gradient and y-intercept, determine values for a and b. Include the uncertainties in your answers. You need not be concerned with the units of a and b.
(where: gradient = b, and y-intercept = $\lg a$) [3]

Solution

$b = 0.50 \pm 0.03$

$\because$ $\lg a = y\text{-intercept}$

$\therefore$ $a = 10^{y\text{-intercept}} = 10^{-0.70} = 0.20$

$a_{max} = 10^{-0.65} = 0.22$

$\Delta a = 0.22 - 0.20 = 0.02$

So:

$a = 0.20 \pm 0.02$

Sample Question 2.32 *(P51/M/J/14)*

A student investigates how the reading V on a voltmeter varies with the resistance Q of a resistor connected into a circuit. It is suggested that V and Q are related by the equation:

$$\frac{V}{E} = -\frac{R}{Q} - \frac{R}{P}$$

where E is the e.m.f. of the cell, and P and R are the resistances of two other resistors connected into the circuit. A graph is plotted of V/E on the y-axis against $1/Q$ on the x-axis. The gradient of the graph is found to be $-640 \pm 110\ \Omega$, and the y-intercept, -0.86 ± 0.38.

(i) Using the values of gradient and y-intercept, determine the values of P and R. Include appropriate units.
(where: gradient = $-R$, and y-intercept = $\frac{-R}{P}$) [2]

(ii) Determine the percentage uncertainty in P. [1]

Solution

(i) $R = -\text{ gradient} = 640\ \Omega$

$$P = \frac{-R}{(y\text{-intercept})} = \frac{-640}{-0.86} = 740\ \Omega$$

(ii) $$\frac{\Delta P}{P} \times 100 = \left(\frac{\Delta R}{R} \times 100\right) + \left(\frac{\Delta(y\text{-intercept})}{(y\text{-intercept})} \times 100\right)$$

$$= \left(\frac{110}{640} \times 100\right) + \left(\frac{0.38}{0.86} \times 100\right) = 61\%$$

Sample Question 2.33 *(P53/O/N/12)*

A student investigates how the maximum speed v of a trolley varies with its total mass M. It is suggested that v and M are related by the equation:

$$v = A\sqrt{\frac{k}{M}}$$

where A is the length of the card attached to the trolley and k is the spring constant of the springs attached to it. A graph is plotted of v^2 on the y-axis against $1/M$ on the x-axis. The value of spring constant k is found to be 23 ± 30% kg s^{-2}. The experiment is repeated using the same springs and a trolley with total mass 0.75 kg. However, the length A of the card is changed from 0.200 ± 0.005 m to 0.100 ± 0.005 m. Determine the maximum speed v of the trolley in m s^{-1}. Include the absolute uncertainty in your answer. [2]

Solution

$$k = 23 \pm 30\% \text{ kg s}^{-2}$$

$$= 23 \pm \frac{30}{100} \times 23$$

$$= 23 \pm 7 \text{ kg s}^{-2}$$

$$v = A\sqrt{\frac{k}{M}} = 0.100\sqrt{\frac{23}{0.75}} = 0.55$$

$$v_{max} = 0.105\sqrt{\frac{30}{0.75}} = 0.66$$

$$\Delta v = 0.66 - 0.55 = 0.11$$

So:

$$v = 0.55 \pm 0.11 \text{ m s}^{-1}$$

Sample Question 2.34 *(P53/O/N/11)*

A student is investigating how a mass m attached to a trolley by a string affects its velocity v. It is suggested that v and m are related by the equation:

$$v^2 = 2gh\left(\frac{m}{m+M}\right)$$

where M is the mass of the trolley, h is the height from which the mass m is released and g is the acceleration of free fall. In this experiment, the value of M is 0.800 kg, value of h is 0.600 m and values of m range from 0.100 kg to 0.600 kg. A graph is plotted of v^2 on the y-axis against $\frac{m}{m+M}$ on the x-axis, and the value of acceleration of free fall g is found to be 9.3 ± 0.7 m s^{-2}. A 1.00 kg mass is now added to the trolley and the experiment is repeated using the same range of values of m. Determine the **largest** possible value of v, in m s^{-1}, that the trolley will gain, using the relationship given and the value of g determined. Include the absolute uncertainty in your answer. [3]

Solution

$M = 1.00 + 0.800 = 1.80$ kg

For the largest possible value of v:

$m = 0.600$ kg

$$v = \sqrt{2gh\left(\frac{m}{m+M}\right)} = \sqrt{2(9.3)(0.600)\left(\frac{0.600}{0.600+1.80}\right)} = 1.67$$

$$v_{max} = \sqrt{2(10.0)(0.600)\left(\frac{0.600}{0.600+1.80}\right)} = 1.73$$

$\Delta v = 1.73 - 1.67 = 0.06$

So:

$v = 1.67 \pm 0.06$ m s^{-1}

Exercise

1 A student investigates how the final velocity v of a cylinder rolling down a board varies with the height h of the board as shown in Fig. 1.1.

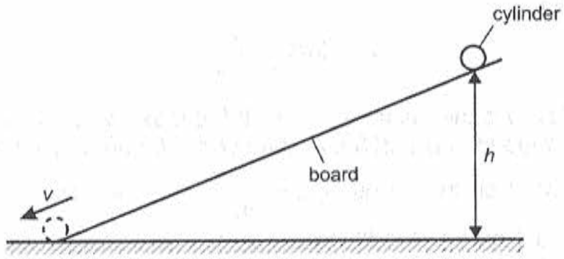


Figure 1.1

For different values of h, the velocity v is determined using a light sensor connected to a data logger. It is suggested that v and h are related by the equation:

$$2gh = v^2Z$$

where g is the acceleration of free fall and Z is a constant.

(a) A graph is plotted of v^2 on the y-axis against h on the x-axis. Determine an expression for the gradient in terms of g and Z. [1]

(b) Values of h and v are given in Fig. 1.2.

h / m	v / m s^{-1}	
0.230	1.40 ± 0.05	
0.280	1.55 ± 0.05	
0.320	1.65 ± 0.05	
0.360	1.75 ± 0.05	
0.400	1.85 ± 0.05	
0.450	1.95 ± 0.05	

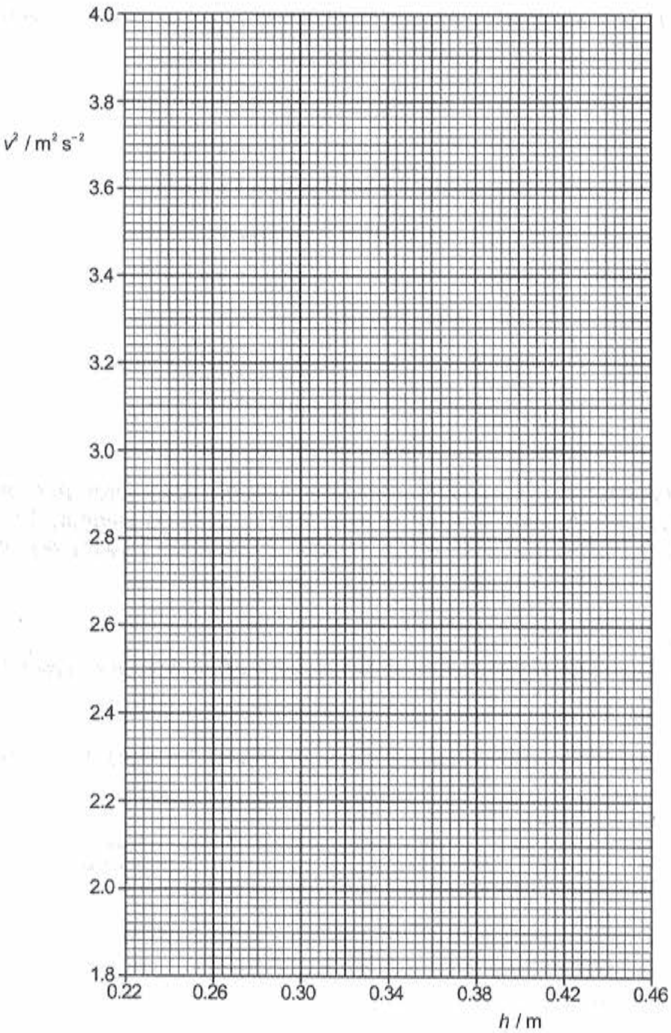
Figure 1.2

Calculate and record values of v^2 / m^2 s^{-2} in Fig. 1.2. Include the absolute uncertainties in v^2.[3]

(c) **(i)** Plot a graph of v^2 / m^2 s^{-2} against h / m. Include error bars for v^2. [2]

(ii) Draw the straight line of best fit and a worst acceptable straight line on your graph. Both lines should be clearly labelled. [2]

(iii) Determine the gradient of the line of best fit. Include the uncertainty in your answer. [2]



(d) The experiment is repeated with h = 0.700 m.

(i) Using your answer to **(c)(iii)**, determine the value of v, in m s^{-1}, using the relationship given. [1]

(ii) Determine the percentage uncertainty in the value of v. [1]

(e) The constant Z is given by:

$$Z = \left(1 + \frac{K}{mr^2}\right)$$

where m is the mass of the cylinder and r is the radius of the cylinder. Using your answers to **(a)** and **(c)(iii)**, determine the value of K. Include the absolute uncertainty in your value and an appropriate unit. Data: g = 9.81 m s^{-2}, m = 2.5 kg and r = 0.015 m. [3]

(P53/O/N/14)

2 A student is investigating how the discharge of a capacitor through a resistor depends on the resistance of the resistor. The equipment is set up as shown in Fig. 2.1.

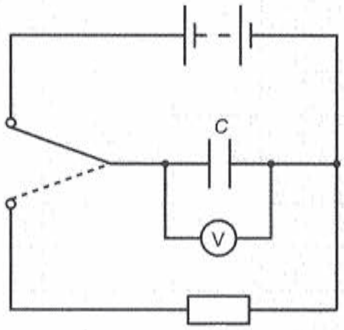


Figure 2.1

The student charges the capacitor of capacitance C and then discharges it through a resistor of resistance R. After 15.0 s the student records the potential difference V across the capacitor. The student repeats this procedure for different values of R. It is suggested that V and R are related by the equation:

$$V = V_0 e^{-\frac{t}{CR}}$$

where V_0 is the initial potential difference across the capacitor and t is the time over which the capacitor has discharged.

(a) A graph is plotted of ln V on the y-axis against $1/R$ on the x-axis. Express the gradient in terms of C. [1]

(b) Values of R and V for t = 15.0 s are given in Fig. 2.2.

R / kΩ	V / V	$(1/R)$ / 10^{-6} Ω^{-1}	ln (V / V)
6.67	3.6 ± 0.2		
10.0	5.0 ± 0.2		
15.0	6.4 ± 0.2		
20.0	7.2 ± 0.2		
30.0	8.0 ± 0.2		

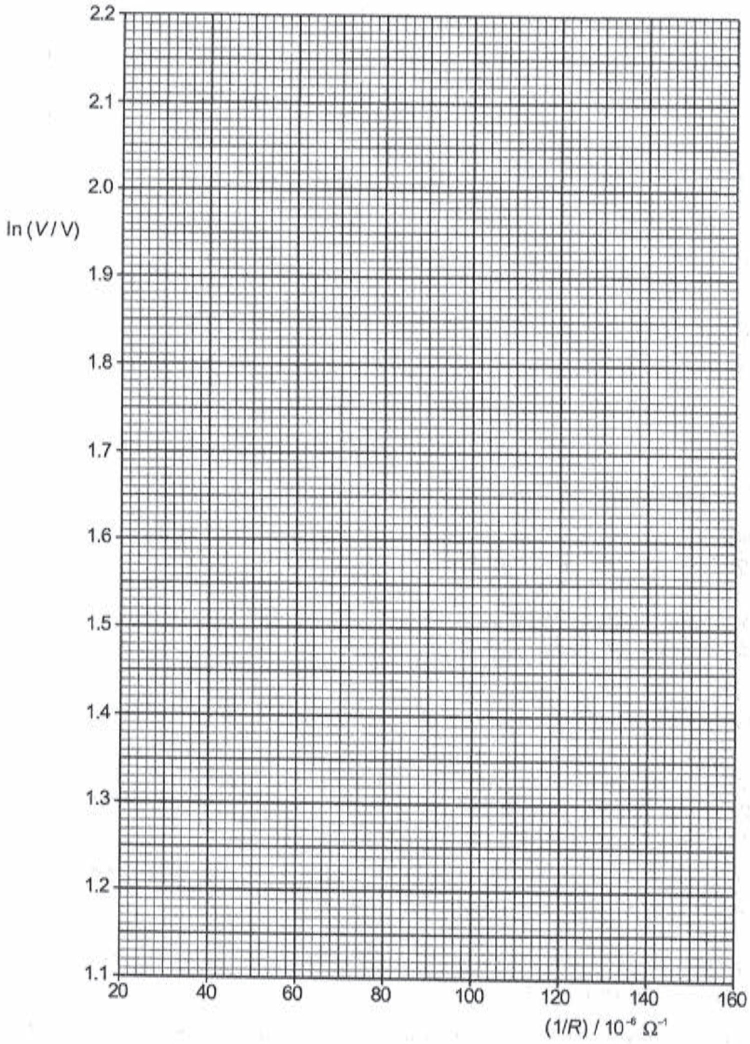
Figure 2.2

Calculate and record values of $(1/R)$ / 10^{-6} Ω^{-1} and ln (V / V) in Fig. 2.2. Include the absolute uncertainties in ln (V / V). [3]

(c) **(i)** Plot a graph of ln (V / V) against $(1/R)$ / 10^{-6} Ω^{-1}. Include error bars for ln (V / V). [2]

(ii) Draw the straight line of best fit and a worst acceptable straight line on your graph. Both lines should be clearly labelled. [2]

(iii) Determine the gradient of the line of best fit. Include the uncertainty in your answer. [2]



(d) **(i)** Using your answer to **(c)(iii)**, determine the value of C. Include an appropriate unit. [2]

(ii) Determine the percentage uncertainty in your value of C. [1]

(e) Determine the value of R, in Ω, for which the capacitor will discharge to 10% of its original potential difference in 15.0 s. Include the absolute uncertainty in your answer. [2]

(P53/O/N/10)

Mark Scheme and Answers

1 **(a)** gradient = $2g/Z$ [1]

(b) v^2 / m^2 s^{-2} [1]

1.96 or 1.960
2.40 or 2.403
2.72 or 2.723
3.06 or 3.063
3.42 or 3.423
3.80 or 3.803

(mixture of significant figures is allowed) [1]

uncertainties increasing from ± 0.1 to ± 0.2 [1]
(more than one significant figures are allowed)

(c) **(i)** All points should be plotted correctly. [1]
(i.e. with an accuracy less than half a small square)

All error bars should be drawn correctly. [1]
(i.e. with an accuracy less than half a small square)

(ii) If points are plotted correctly then lower end of the line of best fit should pass between (0.24, 2.04) and (0.24, 2.10) and upper end of the line should pass between (0.42, 3.54) and (0.42, 3.60). [1]

Worst-acceptable line should be the steepest or shallowest possible line passing through all error bars and should be clearly labelled. [1]

(iii) The triangle used should be at least half the length of the drawn line, and gradient should be about 8.4. [1]

Δ(gradient) = gradient of best-fit line – gradient of worst-acceptable line [1]

(d) **(i)** $v = \sqrt{\text{gradient} \times h}$
should be between 2.39 and 2.46 [1]

(ii) $\dfrac{\Delta v}{v} \times 100 = \dfrac{1}{2}\dfrac{\Delta(\text{gradient})}{(\text{gradient})} \times 100$ [1]

(e) $K = mr^2\left(\dfrac{2g}{\text{gradient}} - 1\right)$
should be between 7.20 × 10^{-4} and 7.80 × 10^{-4} and given to 2 or 3 s.f. [1]

kg m^2 [1]

$\Delta K = K_{max} - K = mr^2\left(\dfrac{2g}{\text{gradient}_{min}} - 1\right) - K$ [1]

2 **(a)** gradient = $-t/C$ [1]

(b)

150	1.28 or 1.281
100	1.61 or 1.609
66.7	1.86 or 1.856
50.0	1.97 or 1.974
33.3	2.08 or 2.079

(mixture of significant figures is allowed) [2]

uncertainties decreasing from ± 0.05 or ± 0.06 to ± 0.02 or ± 0.03 [1]
(more than one significant figures are allowed)

(c) **(i)** All points should be plotted correctly. [1]
(i.e. with an accuracy less than half a small square)

All error bars in ln (V / V) should be drawn correctly. [1]
(i.e. with an accuracy less than half a small square)

(ii) If points are plotted correctly then lower end of the line of best fit should pass between (160, 1.20) and (160, 1.225) <u>and</u> upper end of the line should pass between (20, 2.16) and (20, 2.18). [1]

Worst-acceptable line should be the steepest or shallowest possible line passing through <u>all</u> error bars <u>and</u> should be clearly labelled. [1]

(iii) The triangle used should be at least half the length of the drawn line <u>and</u> gradient should be negative. [1]

Δ(gradient) = gradient of best-fit line – gradient of worst-acceptable line [1]

(d) **(i)** $C = -15$/gradient
should be between 2.14×10^{-3} and 2.24×10^{-3} <u>and</u> given to 2 or 3 s.f. [1]

s Ω^{-1} or F [1]

(ii) $\frac{\Delta C}{C} \times 100 = \frac{\Delta(\text{gradient})}{(\text{gradient})} \times 100$ [1]

(e) $R = 6.514/C$
should be about 3000 Ω [1]

$\Delta R = R_{max} - R = \frac{6.514}{C_{min}} - R$ [1]