

Force: Rate of change of momentum

Force can cause object to:

Shear, Bend, Stretch, compress etc.

Extension: Change in length due to application of force.

$x$  = Stretched length - original length.

$x$  = -ve length decreases

$x$  = +ve length increases.

Hooke's Law: Force is directly proportional to extension until limit of proportionality is reached.

$$F \propto x$$

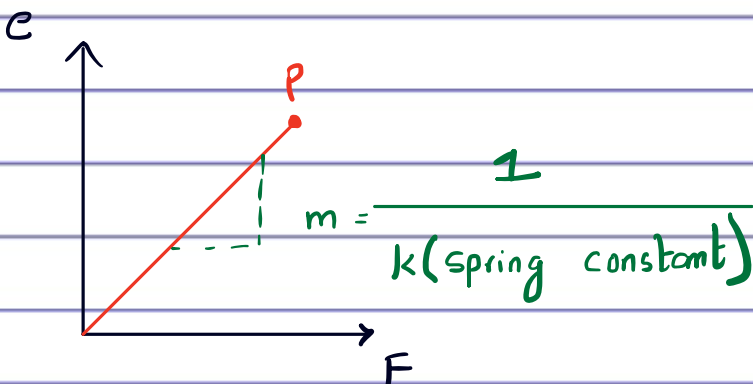
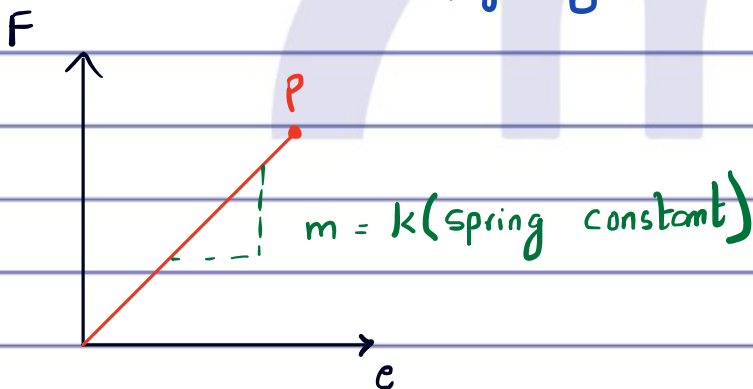
$$F = kx$$

$k$  = Spring constant

$$\therefore \text{Nm}^{-1}$$

How to convert  $\text{Ncm}^{-1}$  to  $\text{Nm}^{-1}$

multiply by 100 [to Convert  $\text{Ncm}^{-1}$  into  $\text{Nm}^{-1}$ ]



Spring constant tells us about the stiffness of the body

Value of  $k$  depends upon design of object and material.

Spring A (Rigid/Stiff spring)

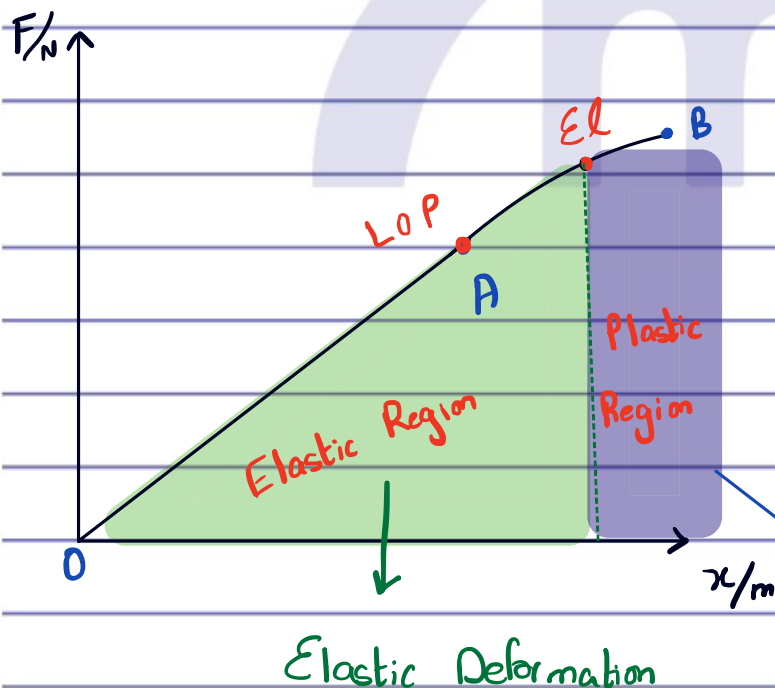
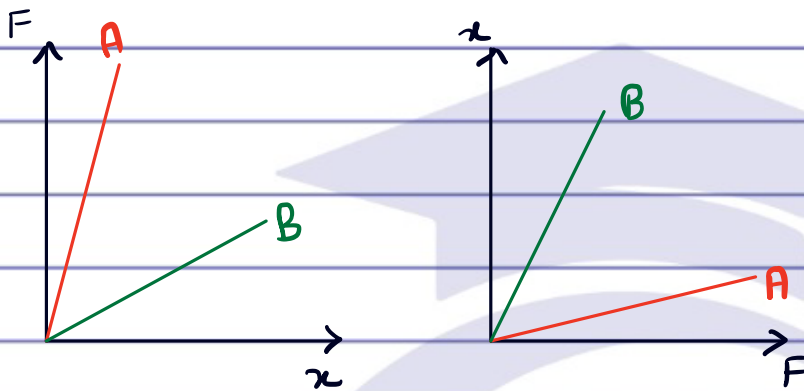


$K_A = 1020 \text{ Nmm}^{-1}$  [means 1020 N force required for 1mm extension]  
 More value of spring constant means more stiffness.

Spring B (flexible)



$K_B = 300 \text{ Nmm}^{-1}$



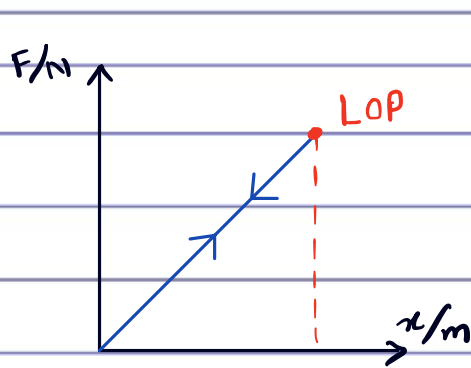
LOP (Limit of Proportionality)

Point till which  $F \propto x$

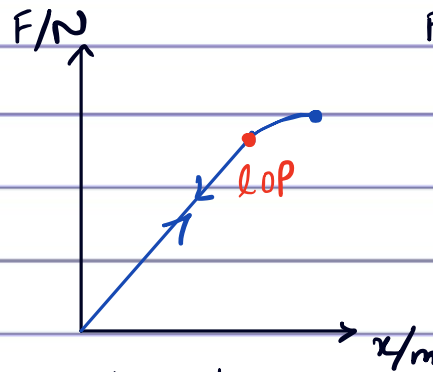
$F-x$  graph is a straight line.

EL (Elastic Limit): Point beyond which permanent deformation occurs.

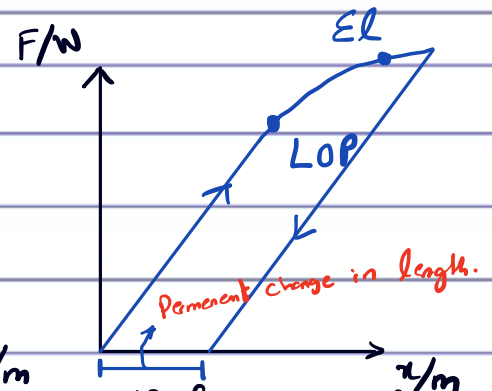
Plastic Deformation.



If force is applied till LOP object returns to its original length



Still returns as Elastic limit is not reached.



If force applied beyond elastic limit permanent deformation occurs.

## Concept of Energy stored by a spring:

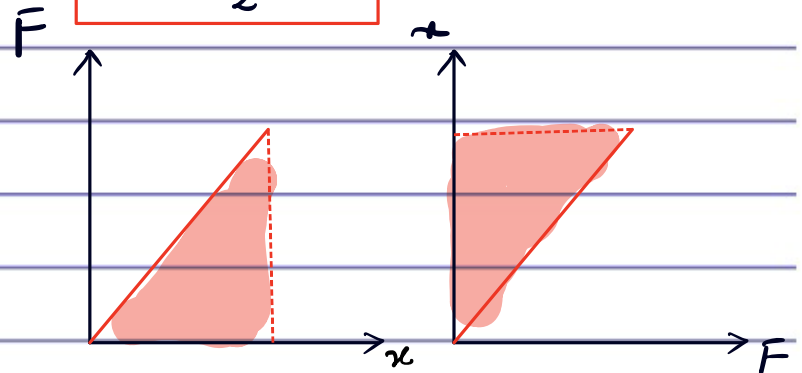
Whenever spring is either stretched or compressed, it stores energy.

\* This energy is called Elastic Potential Energy, strain energy or workdone by the spring.

This energy can be obtained by area of  $F-x$  graph

Energy stored in an object due to deformation.

$$EPE = \frac{1}{2} Fx$$

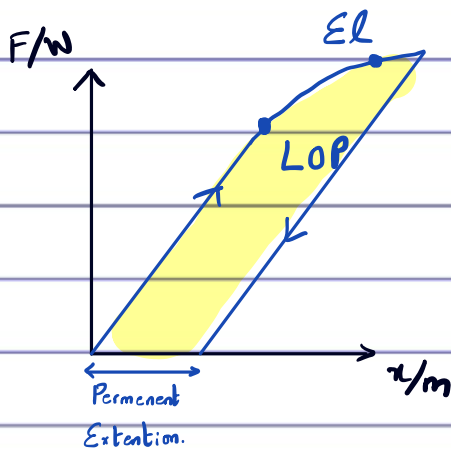


$$EPE = \frac{1}{2} Fx$$

$$\therefore F = kx$$

$$EPE = \frac{1}{2} (kx)(x)$$

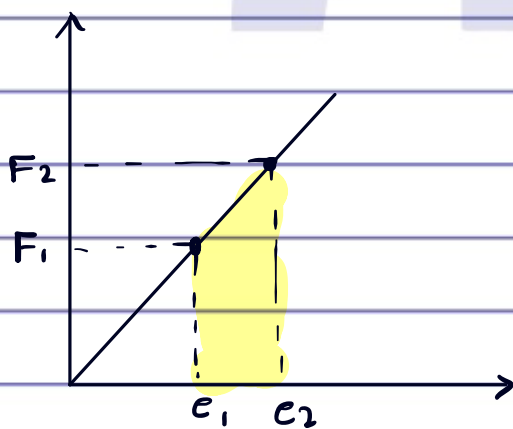
$$\Rightarrow \frac{1}{2} kx^2 = EPE$$



Non Recoverable Energy:  
 EPE stored due to  
 Permanent deformation.  
 (lost in heat)

## "Concept of Additional Strain Energy."

Consider the material which has undergone an initial extension  $e_1$  when a force of  $F_1$  has been applied when force is increased to  $F_2$  and corresponding extension is represented by  $e_2$  then energy stored in second stage is given the name of additional strain Energy.



Show that Additional strain energy is given by  $\frac{1}{2} k (e_2^2 - e_1^2)$

$$\frac{1}{2} (F_1 + F_2) \times (e_2 - e_1)$$

$$F = kx$$

$$\frac{1}{2} (ke_1 + ke_2) (e_2 - e_1)$$

$$\frac{1}{2} k (e_2 + e_1) (e_2 - e_1)$$

$$E = \frac{1}{2} k (e_2^2 - e_1^2)$$

Example no 1 :  $k = 30 \text{ N cm}^{-1}$

$$e_1 = 5 \text{ cm}$$

$$e_2 = 7 \text{ cm}$$

Calculate Additional Strain Energy.

$$\frac{1}{2} k (e_2^2 - e_1^2)$$

$$\frac{1}{2} (3000) (0.07^2 - 0.05^2)$$

$$= +3.6 \text{ J (energy gained)}$$

Example no 2  $k = 40 \text{ N cm}^{-1}$

$$e_1 = 6 \text{ cm}$$

$$\text{to } e_2 = 4 \text{ cm}$$

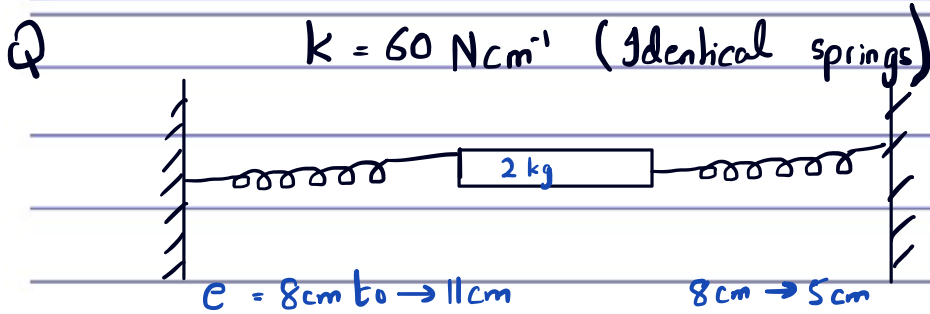
Can we still use the term additional strain energy.

Yes! Although you should get your answer in negative.

What is the significance of -ve answer?

$$\text{Additional Strain Energy} = \frac{1}{2} (4000) (0.04^2 - 0.06^2)$$

$$-4 \text{ J (Energy released)}$$



$$\frac{1}{2} k (e_2^2 - e_1^2)$$

$$\frac{1}{2} (6000) (0.05^2 - 0.08^2)$$

$$\frac{1}{2} (6000) (0.11^2 - 0.08^2)$$

$$-11.7 \quad 17.1 + (-11.7)$$

$$17.1 \text{ J}$$

$$5.4 \text{ J}$$

Given that all this energy is converted to KE of the block calculate initial speed of the block.

$$5.4 = \frac{1}{2} m v^2$$

$$v = 2.3 \text{ m/s}$$

$$5.4 \times 2 = 2(v^2)$$

Describe the motion of this spring.

Oscillatory motion. about a mean position.

Topic: \_\_\_\_\_

$$F = kx$$

$$\frac{F}{k} = x$$

Date: \_\_\_\_\_

Q) Spring P | Spring Q

→ F

→ F

→ k

→ 3k

$$e = \frac{F}{k}$$

$$\rightarrow \frac{F}{3k}$$

$$= \frac{\frac{1}{2} F_1 x_1}{\frac{1}{2} F_2 x_2} = \frac{F \frac{F}{k}}{F \frac{F}{3k}} \quad 1 \div \frac{1}{3} \uparrow$$

Find ratio of strain Energy in P  
Strain Energy in Q

3 : 1

Q) Spring P | Spring Q

5F

7F

3k

8k

$$e = \frac{5F}{3k}$$

$$e = \frac{7F}{8k}$$

$$\Rightarrow \frac{\frac{1}{2}(5F)\left(\frac{5F}{3k}\right)}{\frac{1}{2}(7F)\left(\frac{7F}{8k}\right)}$$

$$\frac{25}{3} \div \frac{49}{8}$$

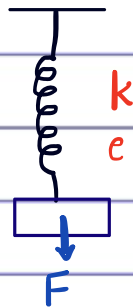
$$\frac{25}{3} \times \frac{8}{49} =$$

Find ratio of Strain Energy in P  
Strain Energy in Q



# Arrangement of Springs (identical) in Series & Parallel Combination

Reference



$$e \propto \frac{1}{k}$$

Series

For identical springs

$$x_T = x_1 + x_2 + x_3$$

(Similar Spring)  $x_T = x_1 + x_2 + x_3$

$$x_T = nx$$

extension of 1 spring.  
no of springs

Total spring constant

$$\frac{1}{k_T} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} \dots$$

$$k_T = \frac{k}{n}$$

$$e_T = 2e$$

$$k_C = \frac{1}{2} k$$

$$e_T = 3e$$

$$k_C = \frac{1}{3} k$$

$$e_T = 4e$$

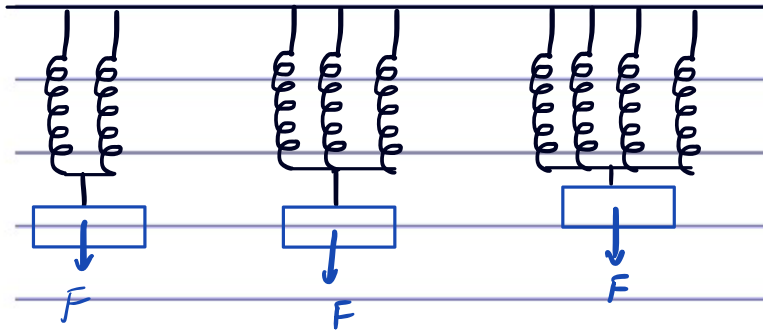
$$k_C = \frac{1}{4} k$$

Each Spring Experience the same force

The Total Extension is always the sum of individual Extension



## Parallel



$$\frac{e}{2}$$

$$2k$$

$$\frac{e}{3}$$

$$3k$$

$$\frac{e}{4}$$

$$4k$$

- Force in Parallel gets divided equally

$$F_{\text{springs}} = \frac{F_T}{n}$$

applied force

no of Springs

Force in each Spring!

$$x_n = \frac{x}{n}$$

extension of one spring

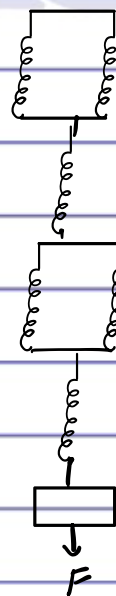
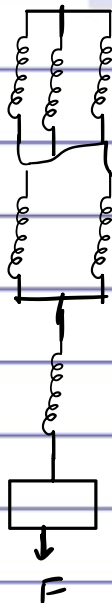
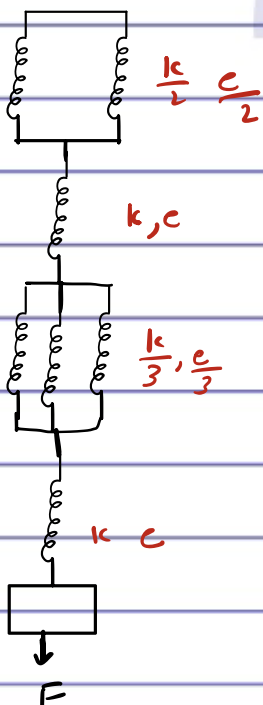
no of Spring.

$$k_n = nk$$

- Total extension is the same as the extension of any one spring

Find the total Extension and combine spring constant. (Identical)

①



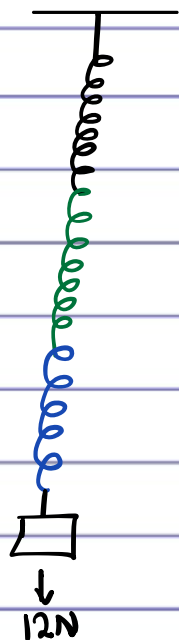
⇒ Find spring constant and extension for these arrangement?

$$F = kx$$

$$\frac{17}{6} \frac{2}{6}$$

$$\frac{6}{17} k$$

a)



$$k = 2 \text{ Nm}^{-1}$$

$$k = 3 \text{ Nm}^{-1}$$

$$k = 6 \text{ Nm}^{-1}$$

→ Calculate Total Extension and  
Combine Spring constant.

$$F = ke$$

$$\frac{12}{6} = e$$

$$2 = e$$

$$F = ke$$

$$\frac{12}{3} = e$$

$$4 = e$$

$$F = ke$$

$$\frac{12}{2} = e$$

$$6 = e$$

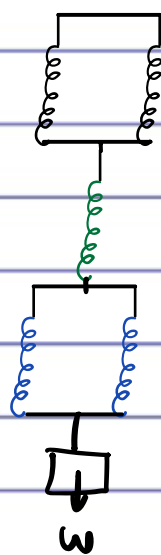
$$F = ke,$$

$$\frac{12}{12} = e$$

$$1 \text{ Nm}^{-1}$$

$$12 \text{ m}$$

b)



$$2k, \frac{W}{4}k$$

$$3k, \frac{W}{3k}$$

$$5k, \frac{W}{10}k$$

Find in terms of  $k$  and  $w$

i) total extension

ii) combine spring constant

$$F = ke$$

$$W = k_e \left( \frac{W}{4}k + \frac{W}{3k} + \frac{W}{10}k \right)$$

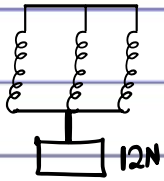
$$\frac{W}{2} = (5k)(e)$$

$$\frac{W}{2} = (2k)(e)$$

$$\frac{W}{10}k$$

$$\frac{W}{4k}$$

## Example 3



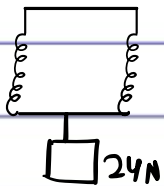
$e = 3\text{cm}$  In the diagram each spring extends by  $3\text{cm}$ .

$$k = ?$$

$$F = ke$$

$$4 = k(3)$$

$$\frac{4}{3} = k$$



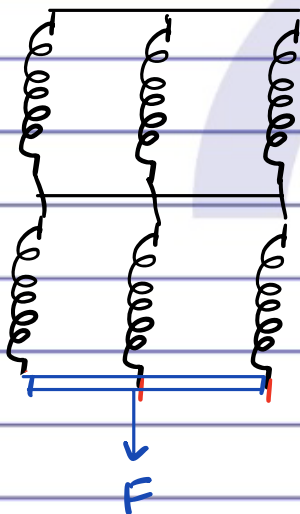
The middle spring is removed and weight is changed to  $24\text{N}$   
calculate New extension.

$$9\text{cm} = e$$

$$12 = \left(\frac{4}{3}\right)(e)$$

$$\frac{36}{4} \quad 9 = e$$

## Example 4



The Diagram shows a spring arrangement state what happens to the total extension if following changes are made independently.

- 1) Increase Number of springs per unit area.

Extension  $\downarrow$   $F$  will be distributed

- 2) Use more layers of springs.

extension will increase.

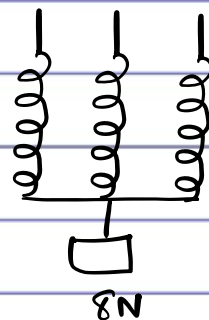
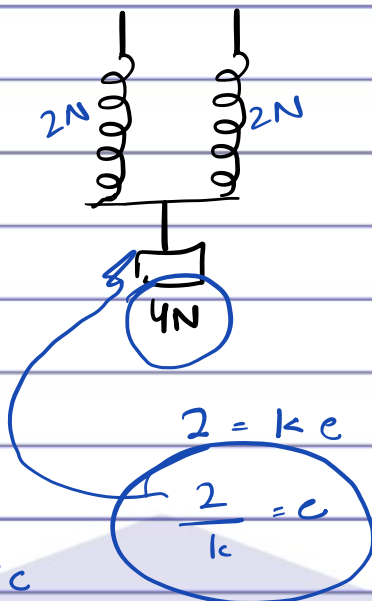
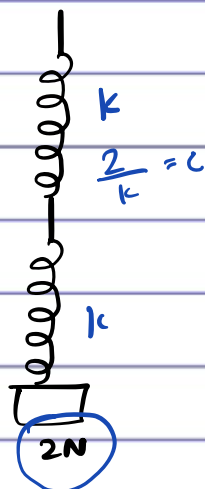
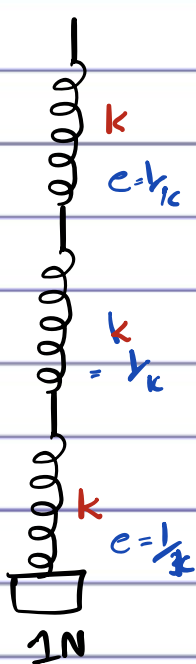
$$F = kce \downarrow$$

$$\uparrow k \propto \frac{1}{e \downarrow}$$

- 3) Replace spring with new with higher spring constant  
ext reduce.

Which option gives greatest extension.

Spring constant =  $k$



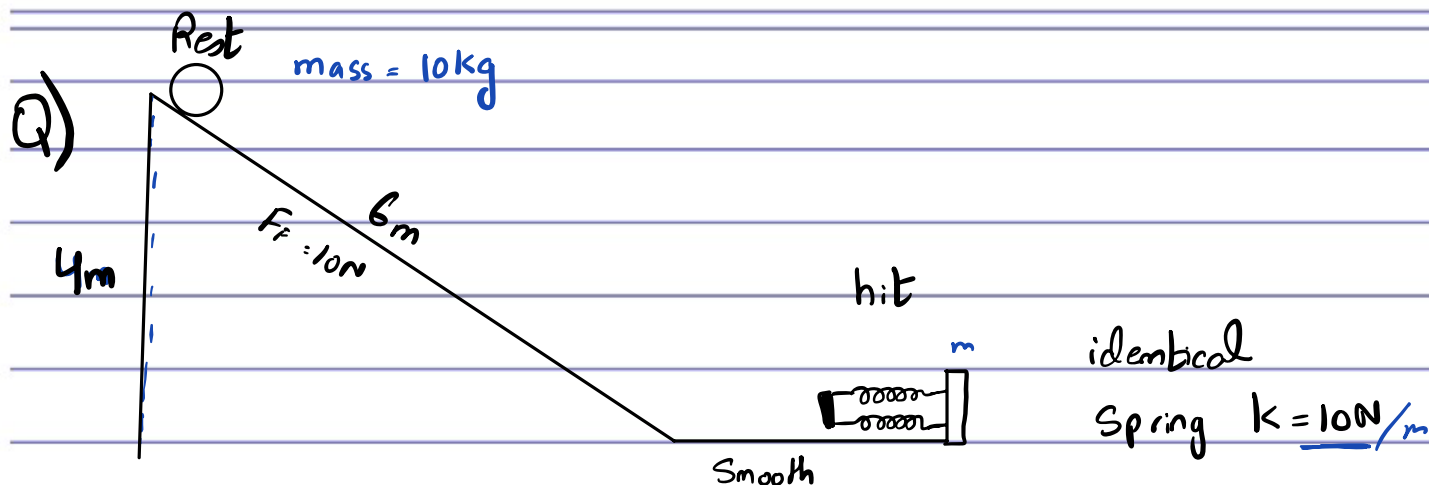
$$F = k e$$

$$\frac{1}{k} = e$$

$$\frac{1}{k} + \frac{1}{k} + \frac{1}{k}$$

$$= \frac{3}{k}$$

$$\frac{4}{k}$$



All KE  $\rightarrow$  EPE calculate compression of Each spring

$$\text{Loss in GPE} - (\text{WD against Friction}) = \frac{1}{2} kx^2$$

$$mgh - F_f d = \frac{1}{2} k(x^2)$$

$$(10)(9.81)(4) - (10 \times 6) = \frac{1}{2} k(x^2)$$

$$392.4 - 60 = \frac{1}{2} (10 \times 2)(x^2)$$

$$\text{Extension } (x) = 5.8\text{ m}$$

Topic: Concept of stress, strain and Young's and Modulus. Date: \_\_\_\_\_

Spring

Wire/Rod.

Force  $\longrightarrow$  Stress

extension  $\longrightarrow$  Strain

Spring constant  $\longrightarrow$  Young's and modulus.

① Stress: Stress is just alternate name of pressure ( $\sigma$ )

units = Pa /  $\text{N/m}^2$

Formula  $\Rightarrow$  
$$\text{Stress} = \frac{\text{Force}}{\text{Area.}}$$



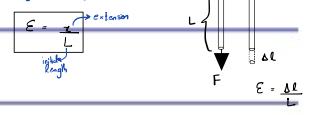
$$\frac{\frac{F}{A}}{\frac{e}{L}} = \frac{FL}{Ac}$$

② Strain = Change in length upon original length

$$\text{Strain} = \frac{\text{change in length}}{\text{original length}} = \frac{\Delta L}{L}$$

Strain: The change in length per unit length.

OR  
Ratio of change in length to original length



extension upon original length.

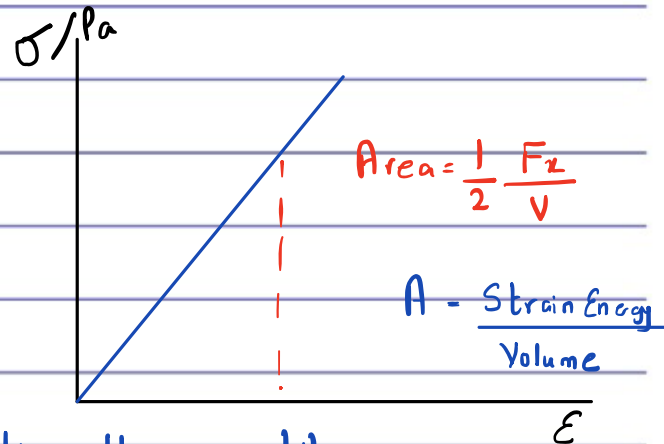
$$\text{Strain} = \frac{e \times L}{OL} \Rightarrow \frac{e}{L}$$

# Youngs Modulus ( $E$ ) The ratio of stress to Strain

$$E = \frac{\sigma}{\epsilon} = \frac{\text{Stress}}{\text{Strain}}$$

$$E = \frac{FL}{A\Delta x}$$

$\Rightarrow$  Pascals



\* Stress and strain helps in making the quantities independent of design.

Stress  $\propto$  strain (limit of Proportionality)

Area under  $\sigma - \epsilon$  graph tells strain energy density i.e. energy stored per unit volume

$$\text{Stress} = E (\text{Strain})$$

$$\epsilon = \frac{\text{Stress}}{\text{Strain}}$$

The more the  $E$  the more Elastic the material (Property of material)

$$E = \frac{FL}{A\Delta x}$$

$$E = \frac{kL}{A}$$

$$F = k\Delta x$$

$$\frac{F}{\Delta x} = k$$

$$E \propto k$$



Rigid material have high value of Youngs Modulus.

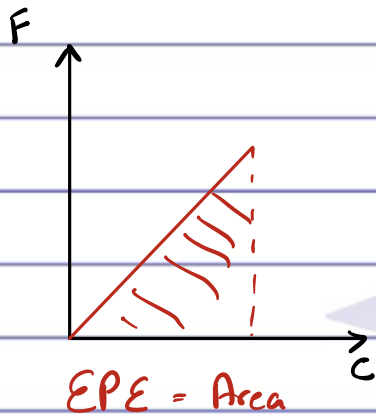
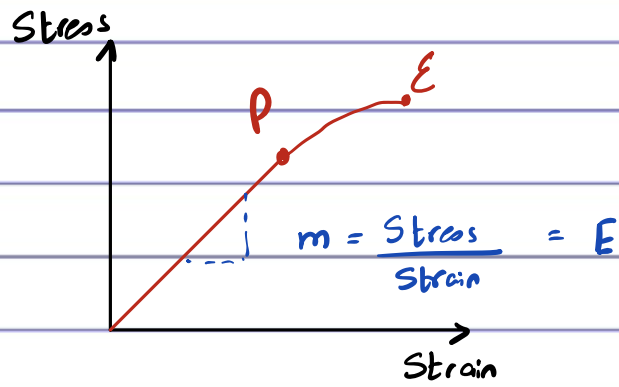
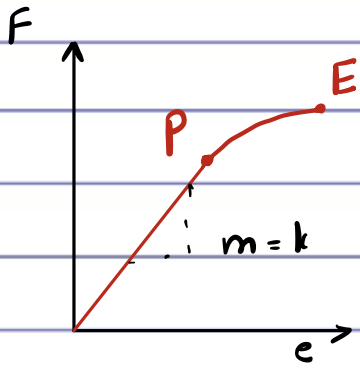


$$\frac{F}{A} \times \frac{e}{L}$$

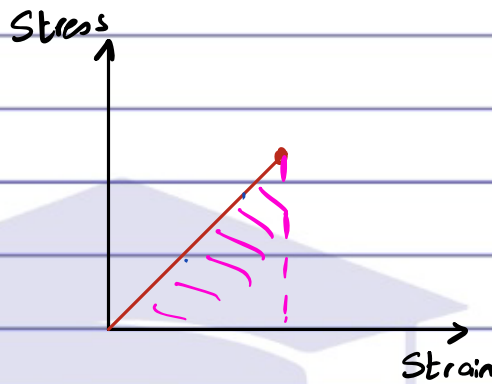
$$\frac{F \times e}{\text{Volume}}$$

Topic: \_\_\_\_\_

Date: \_\_\_\_\_



$$\frac{1}{2} \times F \times e$$



$$\text{Area} = \frac{1}{2} \times \text{Stress} \times \text{Strain}$$

$$= \frac{1}{2} \times \frac{F}{A} \times \frac{e}{L}$$

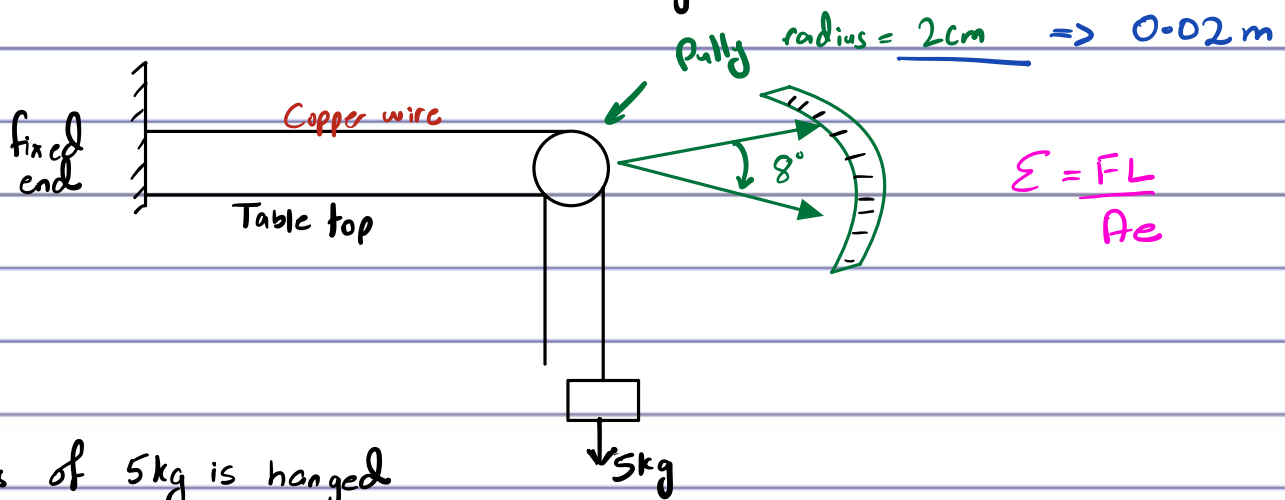
→ Strain energy / EPE

→ Volume.

$$\text{Area} = \frac{\text{Strain Energy}}{\text{Volume.}}$$

Area under stress strain graph  
gives Strain Energy per unit Volume.

# Experiment to determine Young's Modulus



⇒ As mass of 5kg is hanged the pointer deflects (due to extension of the wire) by 8° as shown above.

⇒ What measurements are to be taken:

$$L = 0.85\text{m}$$

$$A = \frac{\pi d^2}{4} = \pi r^2$$

$$F = W = mg. \quad 5(9.81) = 49\text{N} \quad \checkmark$$

To find extension, Lets assume if pulley rotates by 360° then extension = Circumference of the pulley i.e.  $2\pi r$

$$360^\circ \rightarrow 2\pi r$$

$$8^\circ \rightarrow x$$

OR

$$\frac{x}{360} \times 2\pi r \Rightarrow \text{Arc length.}$$

$$e = \frac{8}{360} \times 2\pi(0.02) = 2.8 \times 10^{-3}\text{m}$$

$E = \frac{FL}{Ae}$	$\frac{49 \times 0.85}{A (2.8 \times 10^{-3})}$
---------------------	-------------------------------------------------

What instruments are needed.

- 1)  $L$  = meter rule
- 2)  $A$  = micrometer to measure diameter.
- 3)  $F = W$  = newton meter / spring Balance.
- e) Scale / protractor

Young's Modulus calculation

$$E = \frac{FL}{Ac} = \frac{(4a)(0.85)}{(4.9 \times 10^{-6})(2.8 \times 10^{-3})}$$

$E =$

⇒ Precautions:

- Do preliminary trials to ensure that weight attached does not cause the wire to reach its breaking point.
- After loading the wire the wire must be unloaded. so that it can be checked that the pointer returns back its original position this is done to ensure Elastic limit is not been exceeded.
- Pulley should be oiled to reduce friction.
- Ensure that the wire is taut (i.e it is free from any bends or kinks)

Topic: \_\_\_\_\_

Date: \_\_\_\_\_

Spring constant is also known as force constant

Concept of Percentage. Q19

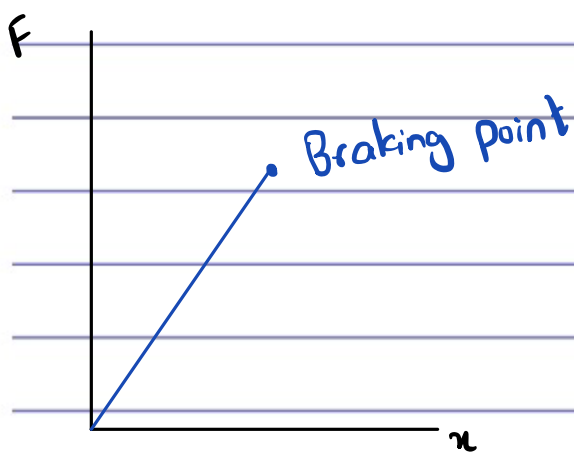


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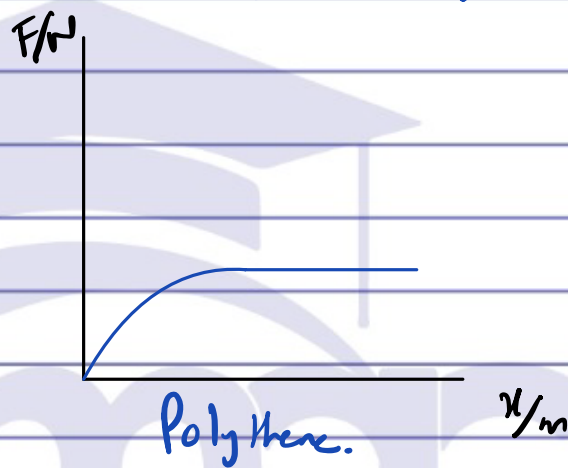
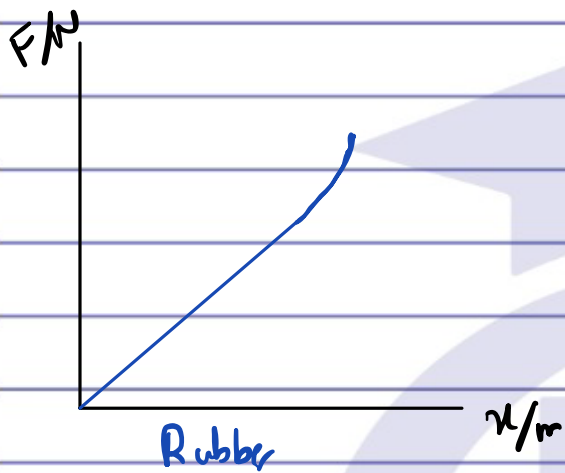
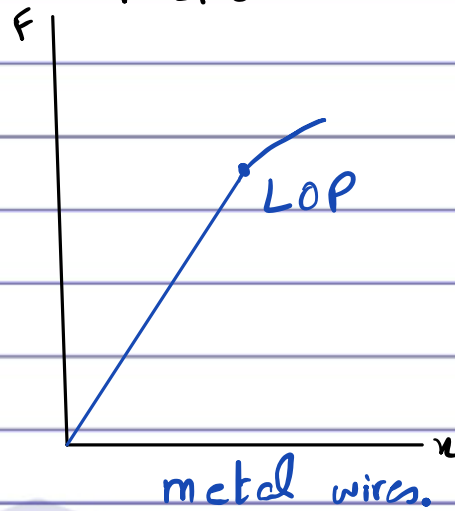
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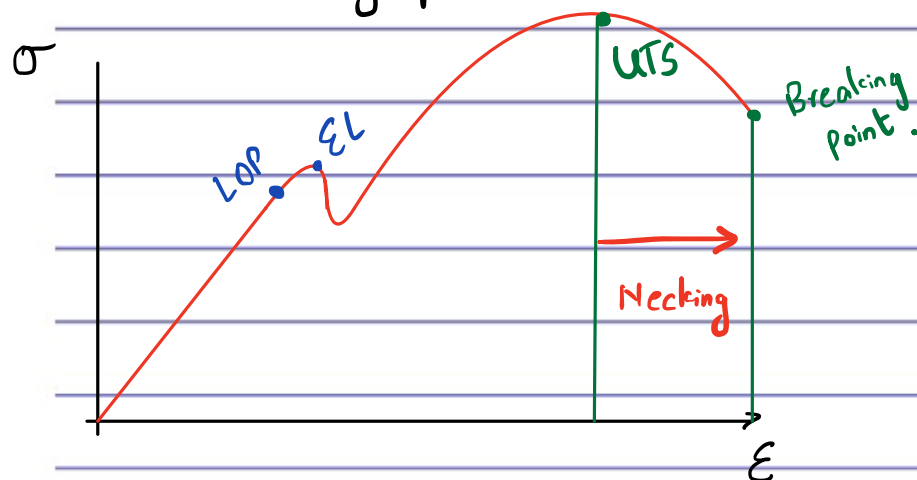
Brittle



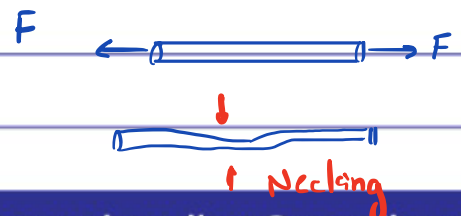
Ductile



Stress - Strain graph of Ductile material.



UTS = (Ultimate Tensile Stress) : The maximum stress a material can endure before failure



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