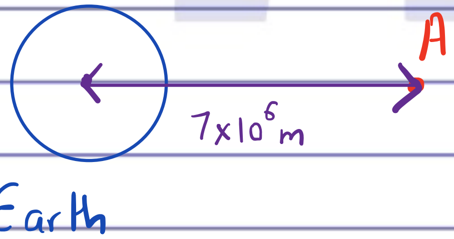
Topic: Gravitational Potential. Date:

Concept of Gravitational Potential.

- Symbol → denoted by symbol "ϕ" phi
- units ⇒ it is measured in J/kg / $J\,kg^{-1}$
- Formula. At a distance r away from the source Mass M, the gravitational potential ϕ can can be obtained using the formula

$$\boxed{\phi = \frac{-GM}{r}}$$

- How do we apply this formula.



Calculate gravitational Potential ϕ at the pt A?

$$\phi = \frac{-(6.67\times10^{-11})(6\times10^{24})}{7\times10^{6}}$$

$$\phi_A = -5.7\times10^{7}\ J\,kg^{-1}$$

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- What information do we get from the result.

The gravitation potential at A tells the amount of Energy lost by a mass of 1 kg, as it moves from infinity to point A within the gravitational field.

The term infinity in this case refers to an assumed distance where gravitational field of Earth is negligible/zero influence.

- How do we define the term. gravitational potential

Gravitational Potential at any point is reffered to amount of workdone in moving a unit mass from infinity to that point in the gravitational field.

* Derivation is not Required

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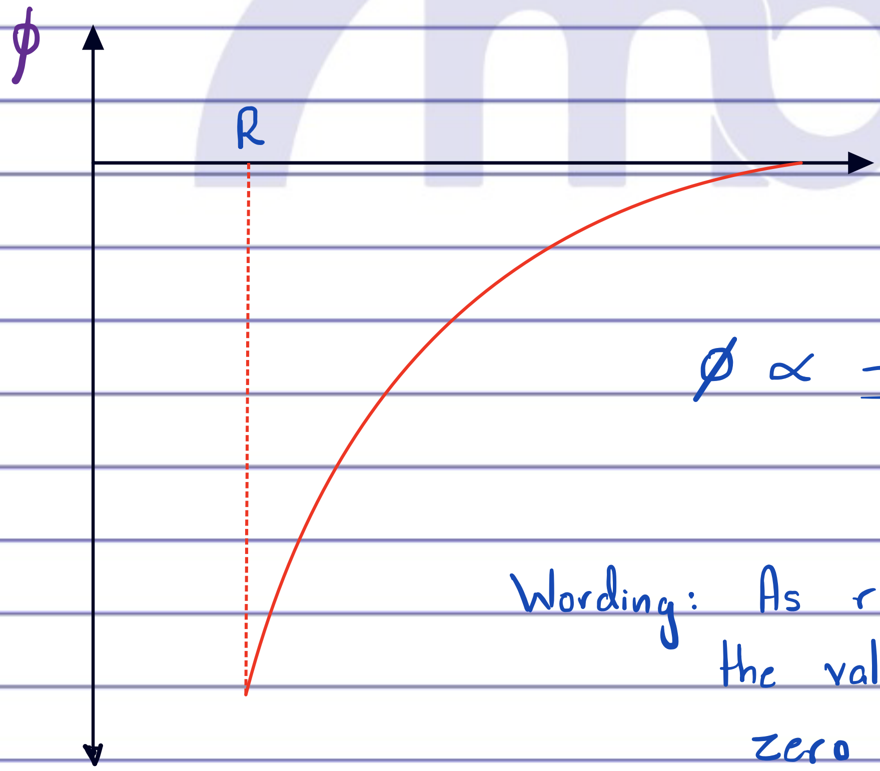
• What is the significance of the minus sign in the formula.

Work is done by the gravitational field in moving the unit mass towards the source mass. Since gravitational field is of an attractive nature ∴ This workdone by the field represents a loss of energy. To signify this loss we insert a negative sign in the formula.

• Graph of Gravitational Potential. $\phi = -\frac{G}{r}$

Graph of ϕ vs r $\uparrow \phi \propto -\frac{1}{r\downarrow}$

$\phi = \frac{-GM}{r}$ {negative Inverse Relationship}



$\phi \propto \frac{-1}{r}$

Wording: As r approches infinity. the value of ϕ approches zero & for all pts within infinity the value of ϕ is negative.

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* Concept of Change in gravitational potential.

Symbol = $\Delta \phi$

Unit = $J\,kg^{-1}$ / J/kg

Formula = $\Delta \phi = \phi_{final} - \phi_{initial}$.

oR

$\Delta \phi = \phi_F - \phi_i$

What does formula imply : The above formula tells us the amount of Energy loss / workdone on the unit mass as it moves from point (Initial position) to another point (final position) within the gravitational field of the source mass.

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Topic: Date:

Concept of Gravitational Potential Energy.

Symbol: "U"

Units: Joule = J

Formula: $\boxed{U = \phi \cdot m \quad , \quad \phi \times m}$

where m is the mass of the moving object

Change in Gravitational Potential Energy.

Symbol: ΔU

Units: "J"

Formula:

$$\Delta U = \Delta\phi \cdot m$$

OR

$$\Delta U = (\phi_F - \phi_i)\, m$$

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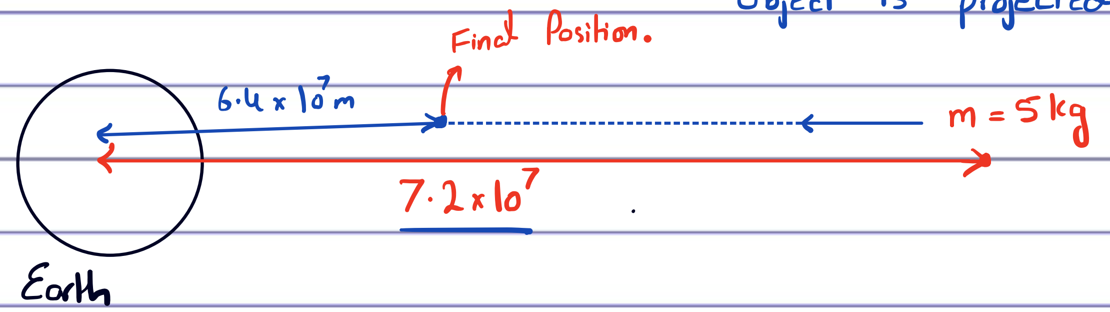
How to apply the above results.

Object is projected inwards

eg. 1



Assuming no Energy loss Calculate the final speed of the object?

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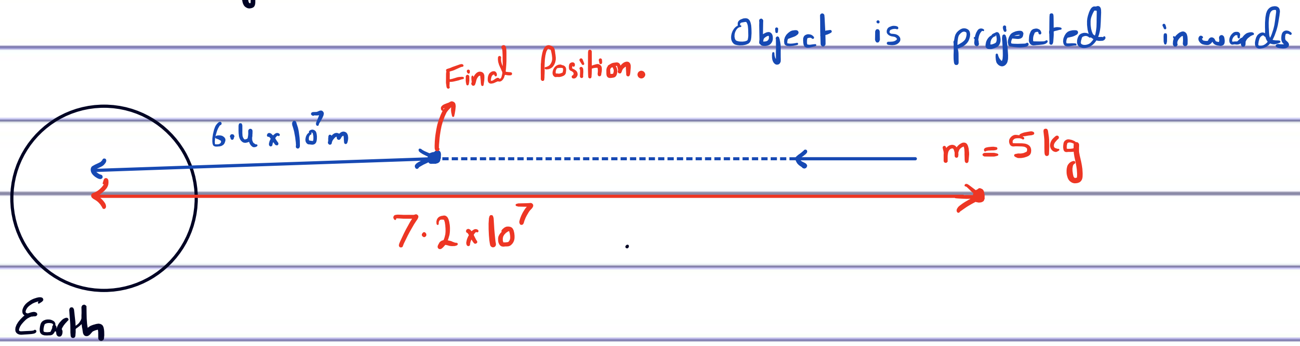
How to apply the above results.

eg.1



Assuming no Energy loss Calculate the final speed of the object ?

* Gain in KE = Loss of GPE (ΔU)

$$\frac{1}{2}\cancel{m}v^2 = \Delta\phi \cdot \cancel{m}$$

$$\frac{1}{2}v^2 = \phi_F - \phi_i$$

$$\frac{1}{2}v^2 = \left(-\frac{GM}{6.4\times10^7}\right) - \left(-\frac{GM}{7.2\times10^7}\right)$$

$G = 6.67 \times 10^{-11}$

$M = 6.0 \times 10^{24}$

$$\frac{1}{2}v^2 = -6.9\times10^5$$

* We can ignore the −ve sign in the final step because in the heading we have already mentioned the term "loss"

$$v = \sqrt{6.9\times10^5 \times 2}$$

$v = 1200\ m/s$ 2S.F

* must give answer in 2SF

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Concept of Escape Velocity. (V_{Esc})

It is defined as minimum speed with which an object can be projected from the earth's surface such that it escapes earth's gravitational field (ie it reaches infinity)

Value of V_{esp} can be calculated using the following Equation (derivation is REQ)

Based on law of conservation of Energy.

Loss of KE = Gain in GPE

$$\frac{1}{2}\cancel{m}(v_E)^2 = \Delta\phi \cdot \cancel{m}$$

* The escape velocity doesn't depend on the mass of the object.

$$\frac{1}{2}(V_{esp})^2 = \phi_F - \phi_i$$

$$\frac{1}{2}(V_{esp})^2 = \left(\frac{-GM}{\infty}\right) - \left(\frac{-GM}{R}\right)$$

→ Anything divided by infinity becomes

→ Any thing divided by zero ...

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* Simplify to get

$$\boxed{V_{esp} = \sqrt{\frac{2GM}{R}}}$$ First Formula

Note that previously we have done $g = \frac{GM}{R^2}$, so

if we make GM the subject, we will get $GM = gR^2$

& if we replace this value in above equation

we get an alternate formula.

$$\boxed{V_{Esp} = \sqrt{2gR}}$$ Second Formula

$G = 6.67 \times 10^{-11}$ $R = 6.4 \times 10^{6}\, m$

$M = 6 \times 10^{24}\, kg$

Solve

$V_{Ecp} = 11200\, m/s$ or $11.2\, km/s$

Conclusion: If Earth is considered to be in isolation then the escape velocity (assuming no energy losses) is 11200 m/s

This principle does not apply to any object which has an Engine / thrust associated with it.

* This is the velocity with which a Baseball will be launched/Projected to escape the Earth's gravitational field.

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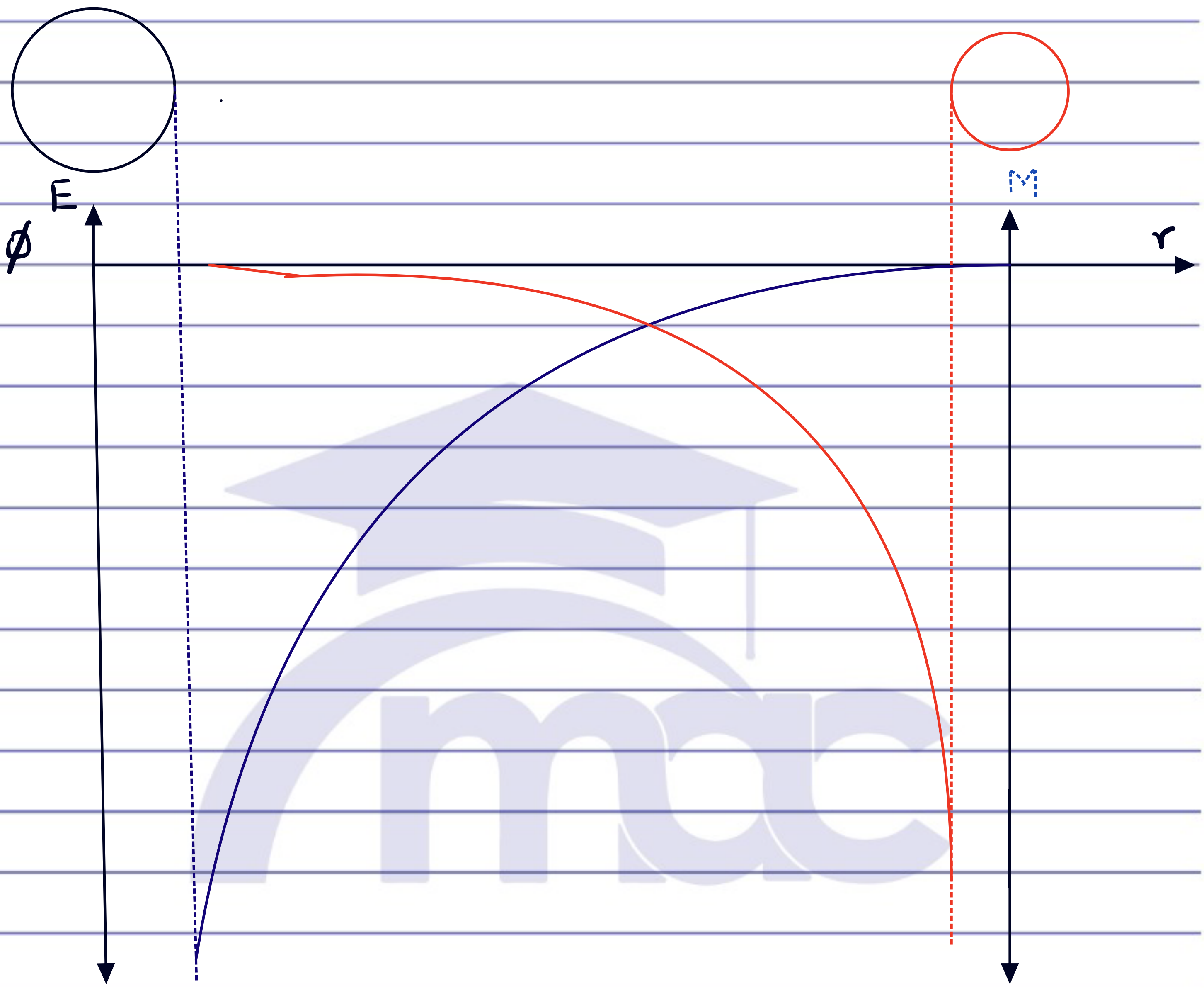
How to calculate the Escape velocity if Earth is not in Isolation if Earth & Moon are both taken into consideration



Sketch the graph of ϕ Res [Compined graph]

On the LHS of null point, Earth's gravitational field is more prominent ∴ The green graph follows the dark blue Pattern

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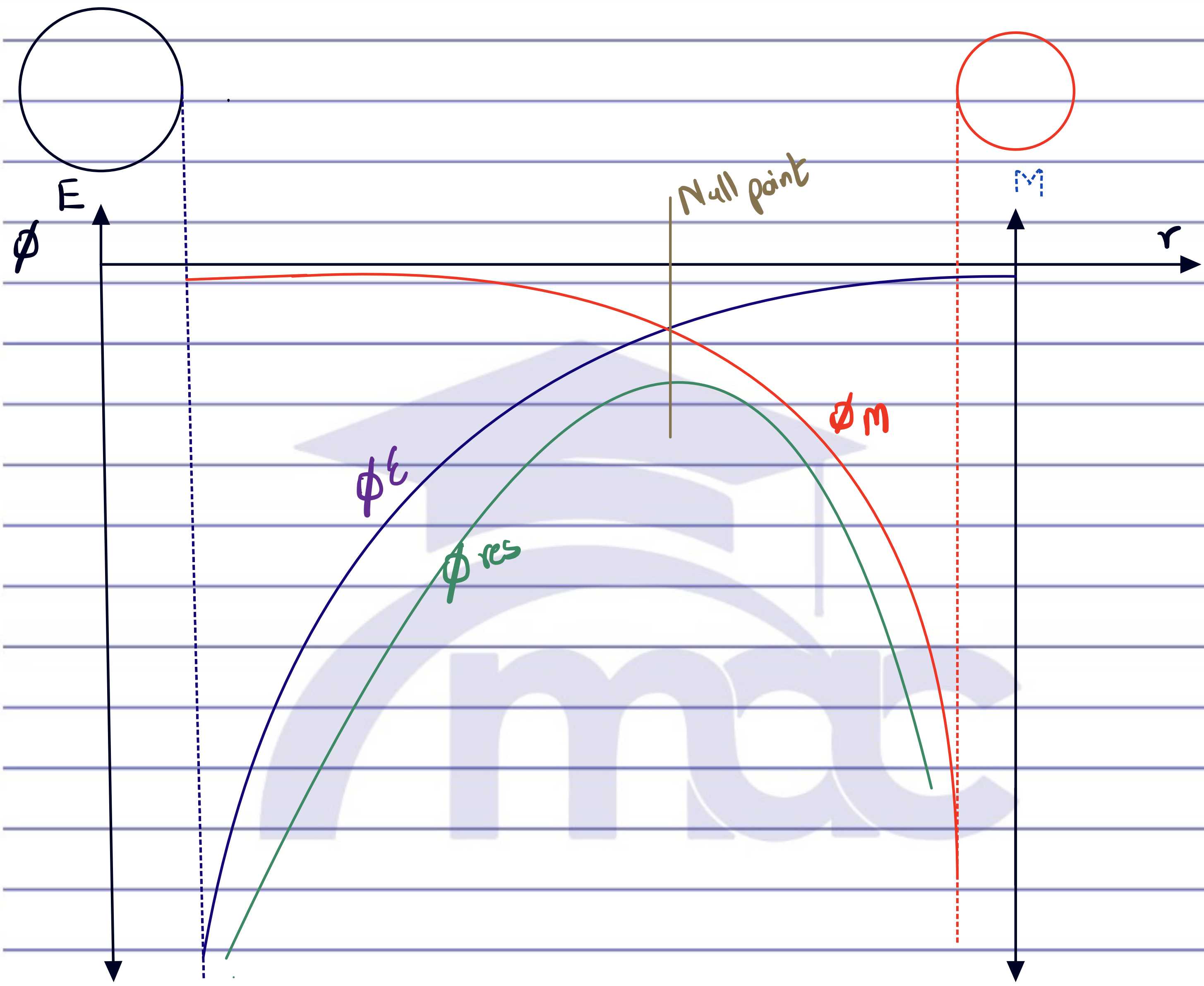
ON RHS the moon gravitational field is more prominent
∴ the green graph follows the red graph



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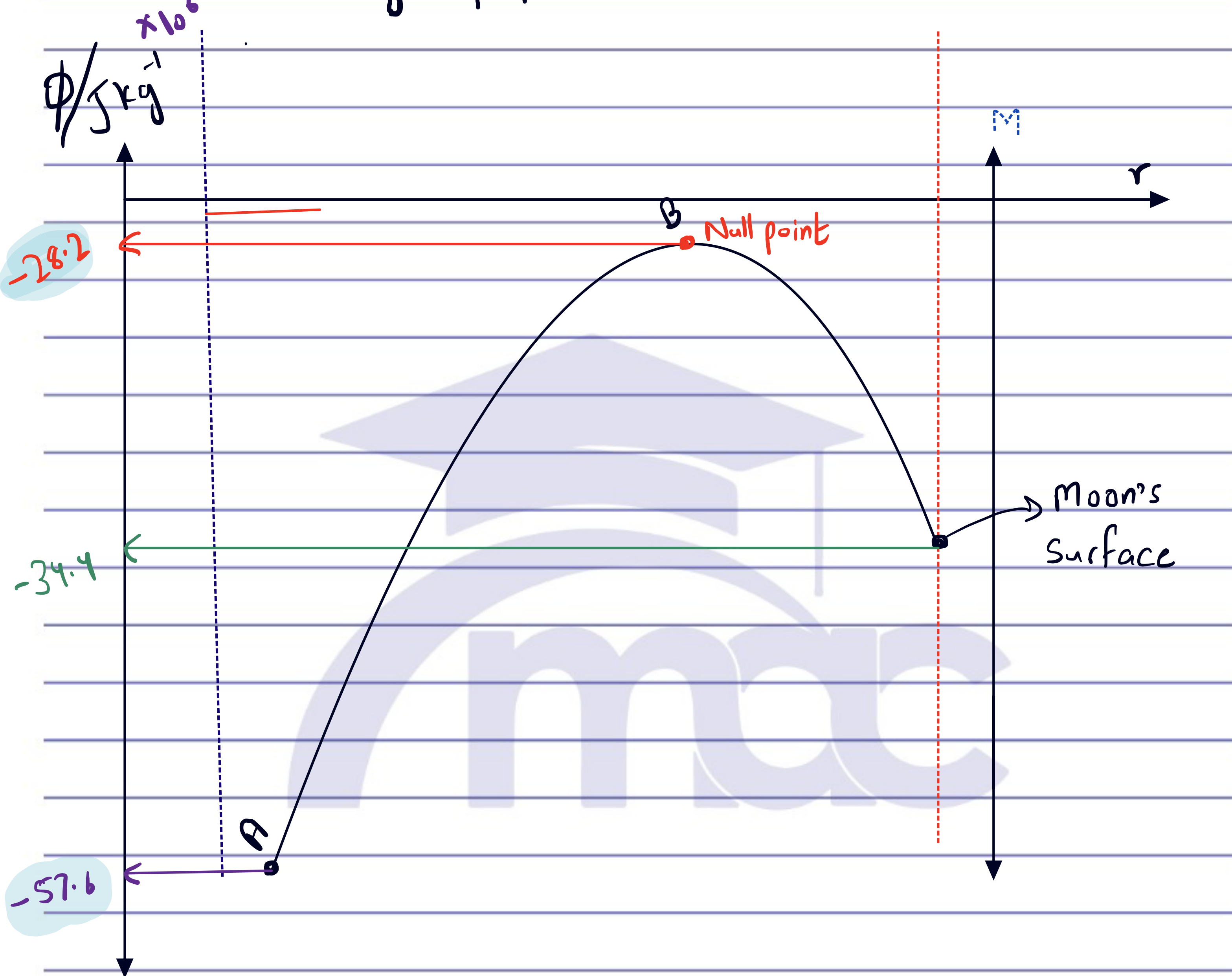
Reconstructing the ϕ Res only with some numerical value for solving purpose



Using given information calculate Escape velocity from earth's surface.

Ans: For object to escape the earth it must be projected with a velocity which is atleast goodenough to insure that it reaches the null point or goes slightly beyond the null point.

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A = initial Position

B = Final Position.

When projected from Earth's surface we can say based on law of conservation of Energy

Loss of KE = Gain in GPE

$$\frac{1}{2}\cancel{m}(V_{Esp})^2 = \Delta\phi\cancel{m}$$

$$\frac{1}{2}(V_{Esp})^2 = \phi_F - \phi_I$$

$$\frac{1}{2}(V_{esp})^2 = \phi B - \phi A$$

$$\frac{1}{2}(V_{Esp})^2 = [-28.2-(-57.6)] \times 10^6$$

Solve to get $V_{esp} = 7700\ m/s$

Q) Why is this value significantly lesser then previous answer of 11,200 (obtained assuming Earth to be in isolation)

Ans) 11200 m/s was required to sent the object all the way to Infinity

This value is much smalle b/c this time we are only projecting the object until it reaches the null point.

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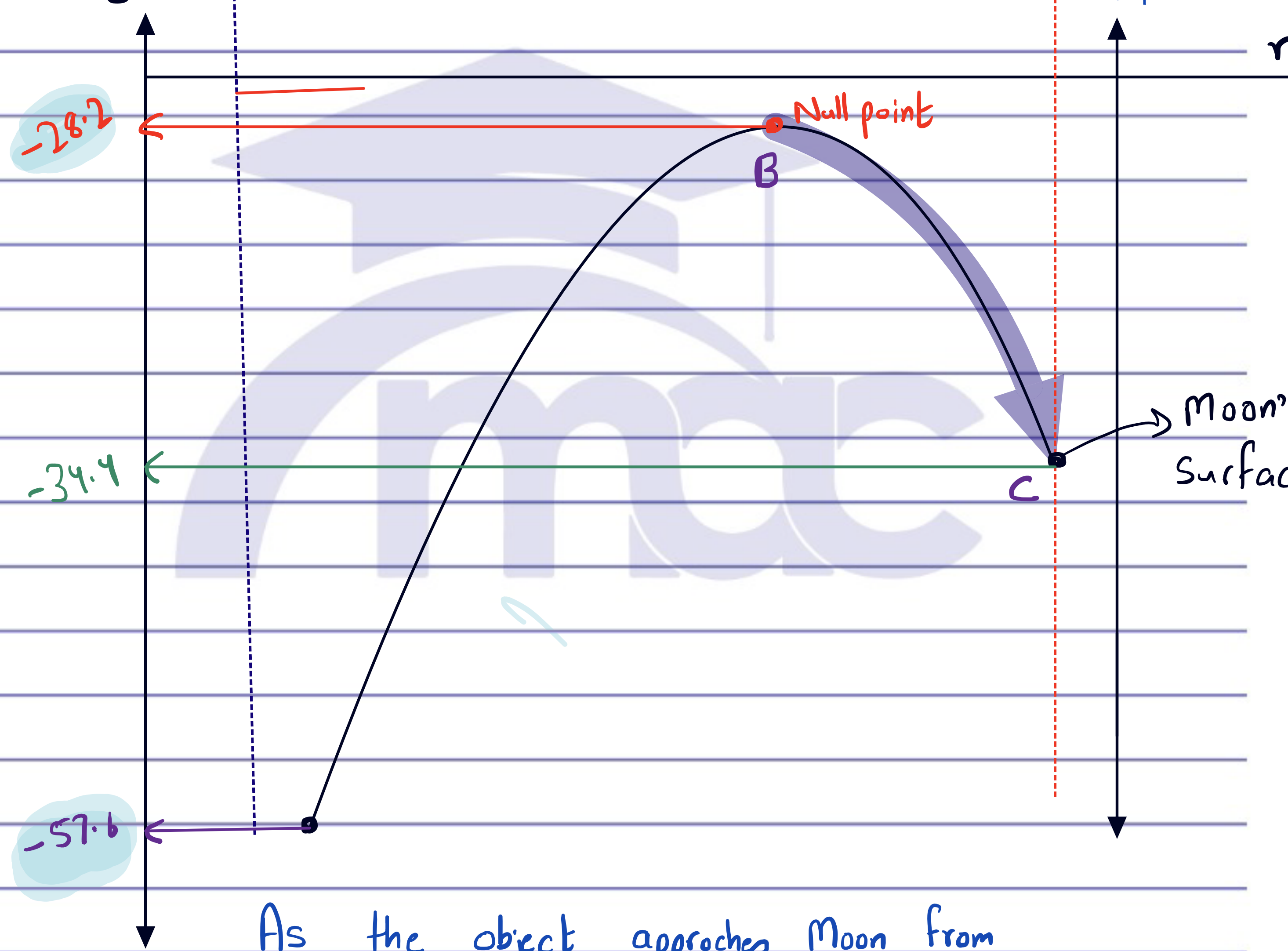
ii) Assuming that it just passes the null point with negligible speed Suggest what will happen?

It will continue moving until it strikes the moon's surface.

iii) Using the above information calculate the velocity/speed with which it strikes the moon.



As the object approches Moon from moon's perspective there is going to be gain in KE and loss in GPE (ΔU)

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$$\frac{1}{2}\cancel{m}\, v_{approach}^{2} = (\phi_F - \phi_i)\cancel{m}$$

$$\frac{1}{2} v^2_{app} = \phi_C - \phi_B$$

$$\frac{1}{2} v^2 = [-34.4 - (-28.2)] \times 10^6$$

2SF

V_{app} as it strikes the moon = 3500 m/s

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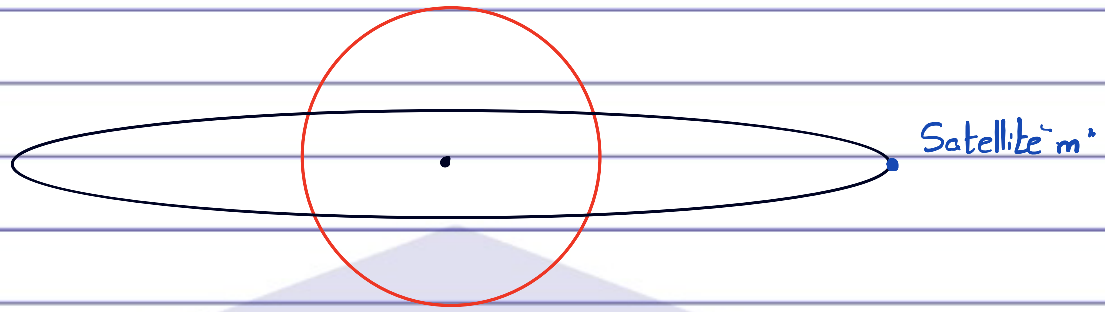
How to calculate Kinetic Energy (KE), Gravitational Potential energy (U) and Total Energy for satellite performing circular motion at a distance "r" away from earth's source mass.



i) $F_c = F_G$

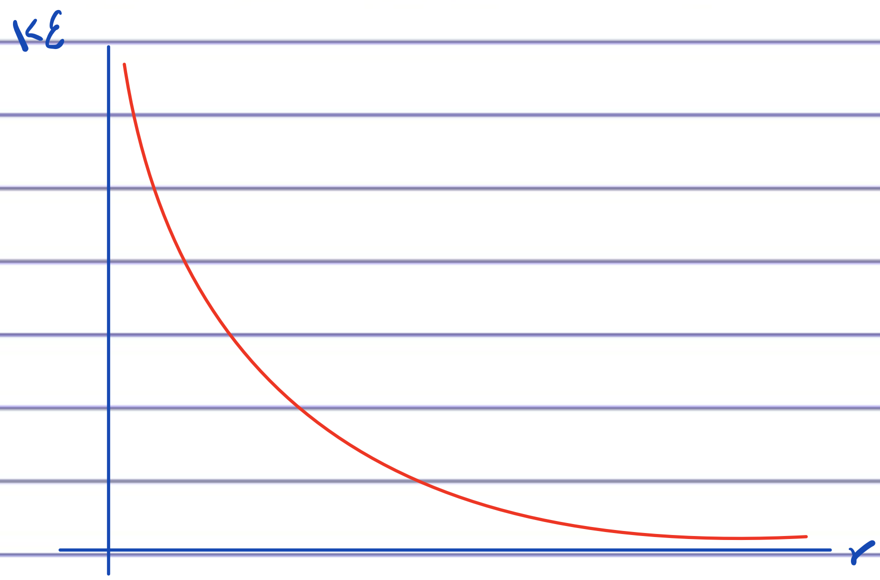
$$\frac{\cancel{m}v^2}{\cancel{r}} = \frac{GM\cancel{m}}{r^{\cancel{2}}}$$

$$\boxed{v^2 = \frac{GM}{r}}$$

$$KE = \frac{1}{2} m v^2$$

$$\boxed{KE = \frac{GMm}{2r}}$$

* Based on Inverse Relationship as radius of the circular orbit increases KE of satellite decreases.



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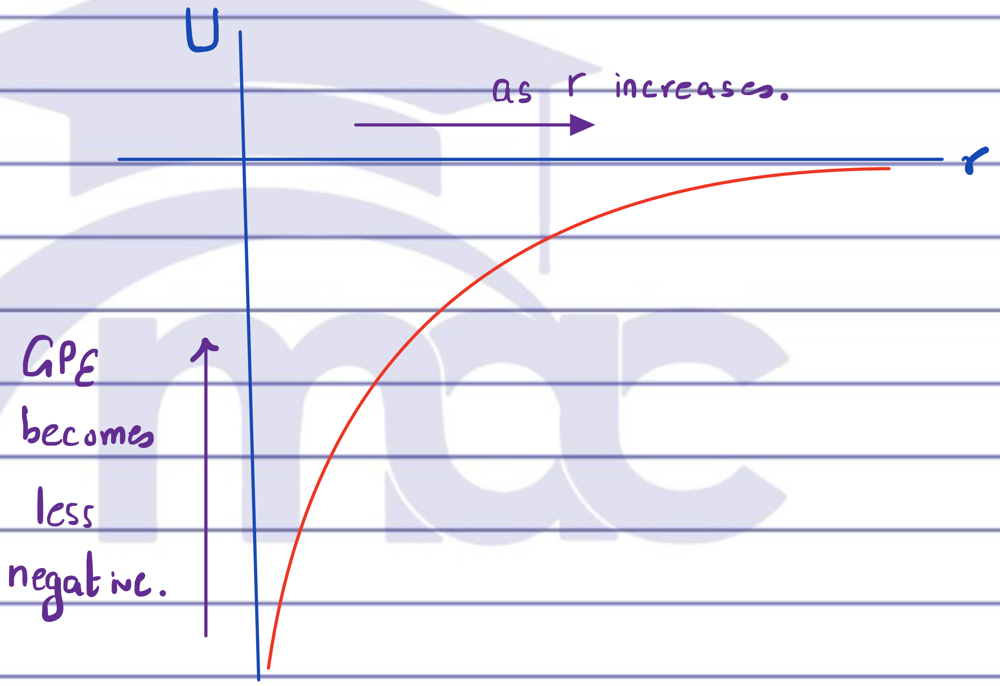
ii) Gravitational Potential Energy (U)

$U = \phi m$

$U = -\frac{GM}{r}(m)$

$U = -\frac{GMm}{r}$ } U – r have negative inverse relationship

Due to negative inverse Relationship as r increases, GPE (U) becomes less negative i.e GPE also increases.



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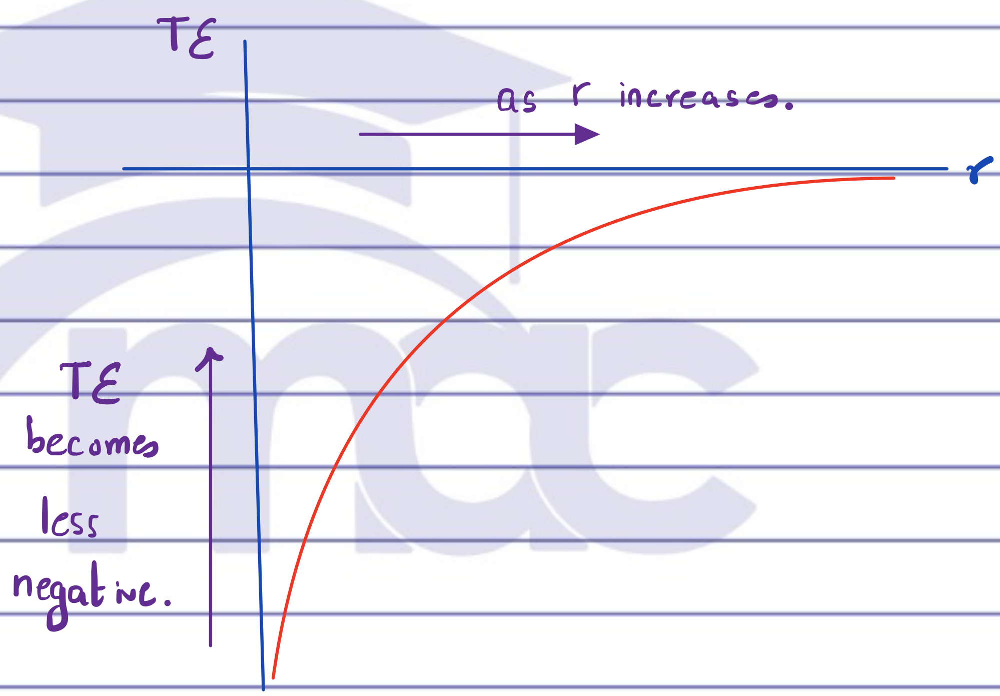
Total Energy

$$TE = KE + U$$

$$TE = \frac{GMm}{2r} + \left(\frac{-GMm}{r}\right)$$

$$TE = \frac{-GMm}{2r}$$

* Due to negative inverse Relationship as r increases TE also increases



Formulas.

$$F_G = \frac{GMm}{r^2}$$

$$g = \frac{GM}{r^2}$$

$$\phi = -\frac{GM}{r}$$

$$\Delta\phi = \phi_F - \phi_I$$

$$U = \phi \cdot m$$

$$\Delta U = \Delta\phi \cdot m$$

$$V_{esc} = \sqrt{\frac{2GM}{R}} \text{ OR}$$

$$V_{esc} = \sqrt{2gR}$$

Satellite

$$V = \sqrt{\frac{GM}{r}}$$

$$\omega = \sqrt{\frac{GM}{r^3}}$$

$$T^2 = \frac{4\pi^2}{GM} \cdot r^3$$

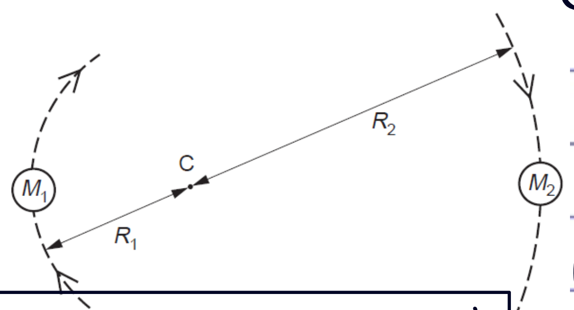
$$K.E = \frac{GMm}{2r}$$

$$U = -\frac{GMm}{r}$$

$$T.E = -\frac{GMm}{2r}$$

Topic: Date:

Concept of Binary Star:



The mass of star M_1 has circular orbit of radius R_1 and star of mass M_2 has a circular radius of R_2 both stars have same angular speed.

(i) Write an Expression for the F_G b/w the two stars

$$F_G = \frac{GM_1M_2}{(R_1+R_2)^2}$$

(ii) Write down an expression for F_c required by M_1 to perform circular motion.

$$F_c = mr\omega^2$$

$$F_c = M_1R_1\omega^2$$

iii) Write down an Expression for F_c required by M_2 to perform circular motion.

$$F_c = mr\omega^2 \qquad F_c = M_2R_2\omega^2$$

iv) Given that F_G provides F_c required by each star to perform circular motion. Use above answers to show.

$$\frac{M_1}{M_2} = \frac{R_2}{R_1}$$

For M_1, $F_G = F_c$

$$\boxed{\frac{GM_1M_2}{(R_1+R_2)^2} = M_1R_1\omega^2} \quad (1)$$

For M_2 $F_G = F_c$

$$\boxed{\frac{GM_1M_2}{(R_1+R_2)^2} = M_2R_2\omega^2} \quad (2)$$

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Equate the results.

$$M_1 R_1 \cancel{\omega^2} = M_2 R_2 \cancel{\omega^2}$$

$$\frac{M_1}{M_2} = \frac{R_2}{R_1} \qquad M \propto \frac{1}{R}$$

hence in Binary star we can say mass of the star is inversly propotional to the radius (R) of its circular orbit

More Massive Star performs circular motion in smaller orbit
Less Massive Star performs circular motion is larger orbit.

Calculations Involving Binary stars.

Information

- Time period = 4 years
- Distance b/w stars = 6.0×10^{10} m
- $M_1 = 4M_2$

$$\frac{G \cancel{M_1} M_2}{(R_1 + R_2)^2} = \cancel{M_1} R_1 \omega^2$$

$$\frac{(6.67 \times 10^{-11})(M_2)}{(6 \times 10^{10})^2} = (1.2 \times 10^{10})\left(\frac{2\pi}{4 \times 365 \times 24 \times 60 \times 60}\right)^2$$

$$\boxed{M_2 = 1.67 \times 10^{27}\ \text{kg}}$$

$\therefore M_1 = 4M_2$

$$\boxed{M_1 = 6.43 \times 10^{27}\ \text{kg}}$$

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Find R_1, R_2

$$\frac{M_1}{M_2} = \frac{R_2}{R_1}$$

$$\boxed{R_2 = 4R_1} \quad (1)$$

$$\frac{4}{1} = \frac{R_2}{R_1}$$

$$\boxed{R_1 + R_2 = 6\times10^{10}} \quad (2)$$

Solve Simultaneously

$$\boxed{R_1 = 1.2\times10^{10}\ m}$$

$$\boxed{R_2 = 4.8\times10^{10}\ m}$$

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