Topic: ⇒ Gravitation

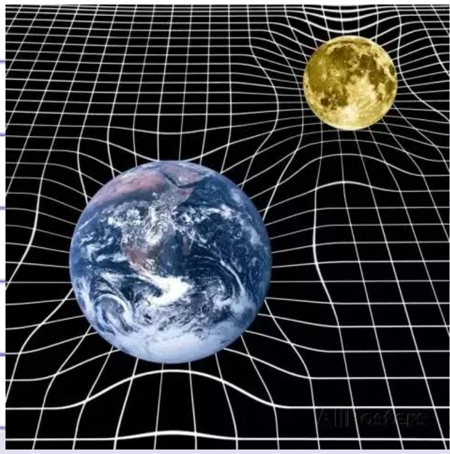
Date:

Gravitation

1st Newton

Einstein.

Bosons

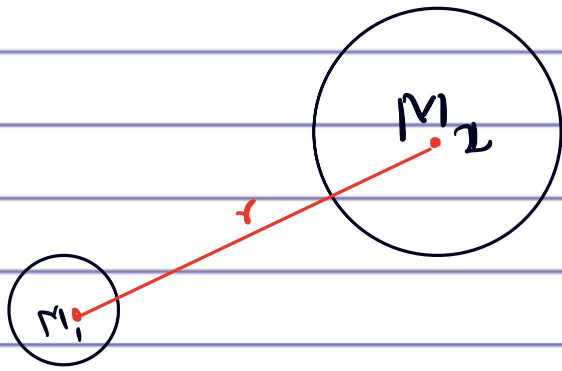


space time Fabric

Newton's law of Gravitation: Acc to newtons law, the gravitational force of attraction (F) or (F_g) b/w the two point masses M_1 & M_2 is known to be directly propotional to the product of the two masses and inversly propotional the square of there distance. b/w their centre.

$$F_G \propto \frac{M_1 M_2}{r^2}$$

$$F_G = \frac{G M_1 M_2}{r^2}$$



Universal Gravitational Constant : $\boxed{G = 6.67 \times 10^{-11}}$

SI unit = $N\ m^2\ kg^{-2}$

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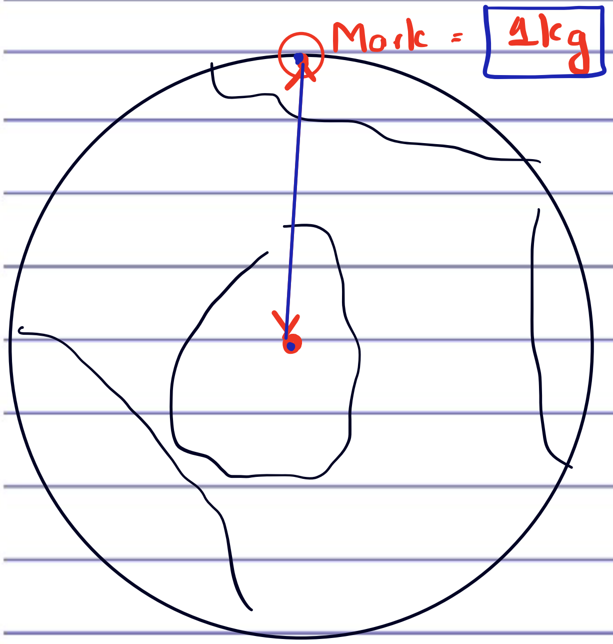

Apply the Gravitational force b/w the marker & earth if the marker is positioned on the surface.

mass of mark = 1kg

$M_E = 6.0 \times 10^{24}$ kg

Radius of Earth = 6.4×10^{6} m

$$F_G = G\frac{M_1 M_2}{(R_E)^2} = \boxed{\frac{(6.67\times10^{-11})(6\times10^{24})(1)}{(6.4\times10^{6})^2}}$$

$$\boxed{F_G = 9.8 N/kg}$$

marker 1kg

$W = mg$

$W = 1 \times 9.8$

$= 9.8$ N/kg

Note: F_G b/w earth and marker. is an alternate name given as the weight of the marker.

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Obtain a relationship b/w g (gravitational field strength) & "G" Universal gravitational constant.

$$\boxed{W = F_G}$$

mass of particle

$$\cancel{m} g = \frac{G M_1 \cancel{m}}{r^2}$$

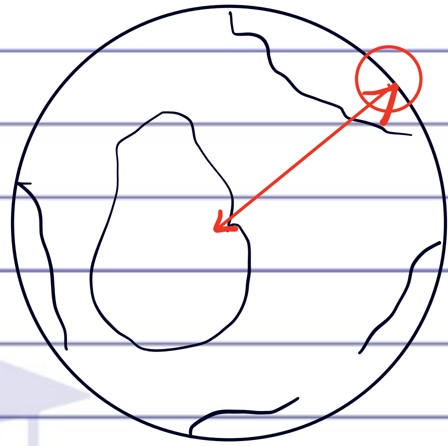
$M_1 \rightarrow$ mass of earth

$m \rightarrow$ mass of particle

$$\boxed{g = \frac{G M_1}{r^2}}$$

W

$$\cancel{m} g = \frac{G M_1 \cancel{m}}{r^2}$$



$$\boxed{g = \frac{G M_1}{r^2}}$$

M = Mass of Source (Earth)

r = radius of planet

$$\boxed{g \propto \frac{1}{r^2}}$$

(Inverse - square relationship)

1/4th

double

$$g = \frac{1}{(3R)^2}$$

$$\frac{1}{(2)^2} = \frac{1}{4}$$

$$g = \frac{1}{9r^2}$$

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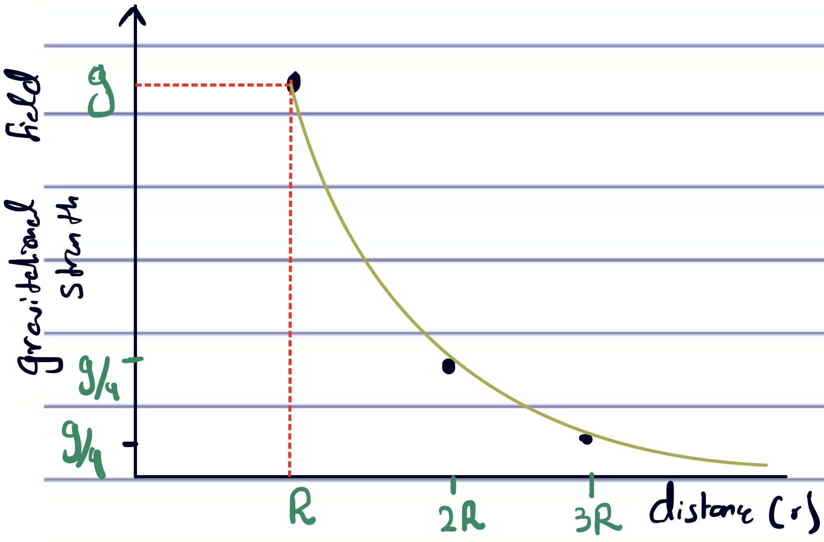


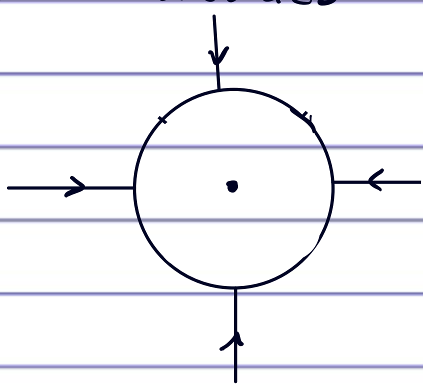
g	R
g	R
1/4	2R
1/9	3R

Every thing shown above is VALID as long as we assume earth to be an isolated body (ie influence of other planetary bodies is not concidered)

⇒ Gravitational field: Region in space where a mass experiences an attractive force.

⇒ Gravitational field strength: force of attraction experienced by a unit mass within the gravitational field.

How to construct gravitational field lines.



gravitational force is only an attractive force hence direction will always be radially inwards. as shown.

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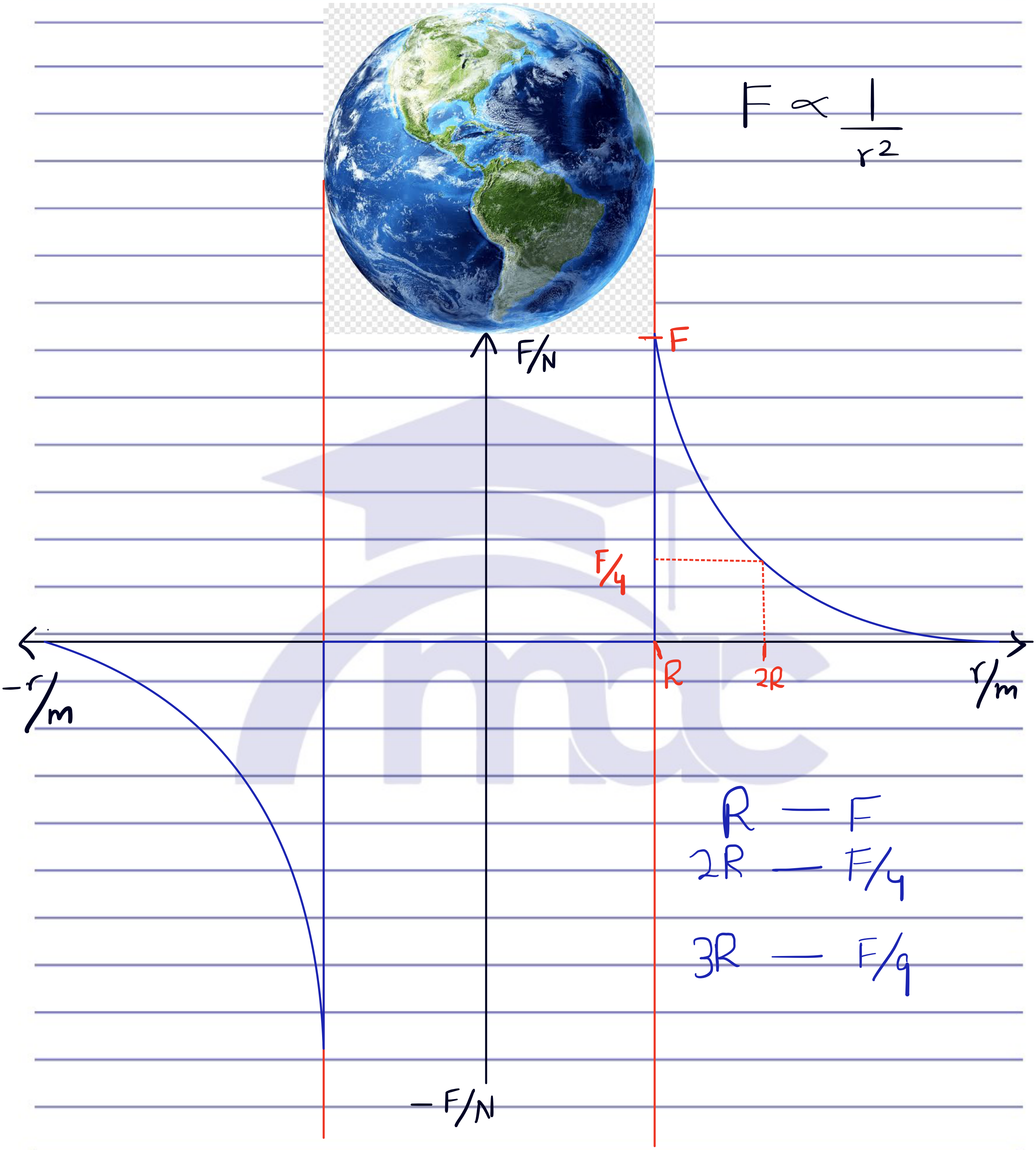
$$F \propto \frac{1}{r^2}$$



R — F

$2R$ — $F/4$

$3R$ — $F/9$

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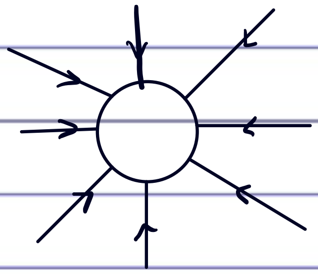
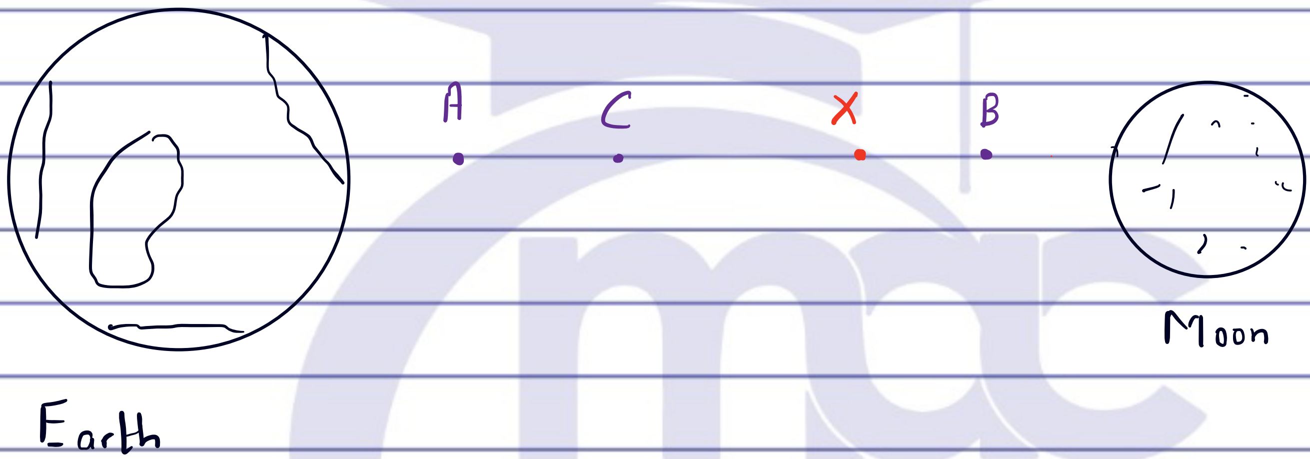
* Same mass but smaller radius.



$$\uparrow g = \frac{G M}{r^2 \downarrow}$$

→ Variations of gravitational field (g) with distance (r) if other planetary bodies eg. Moon is taken into consideration



For A

$g_E > g_m$

For B

$g_m > g_E$

For C

$g_E > g_m$

For D

$g_E = g_m$

Particle at x might remain stationary.
g_E balances out with g_m (Null point)
g resultant $= 0$

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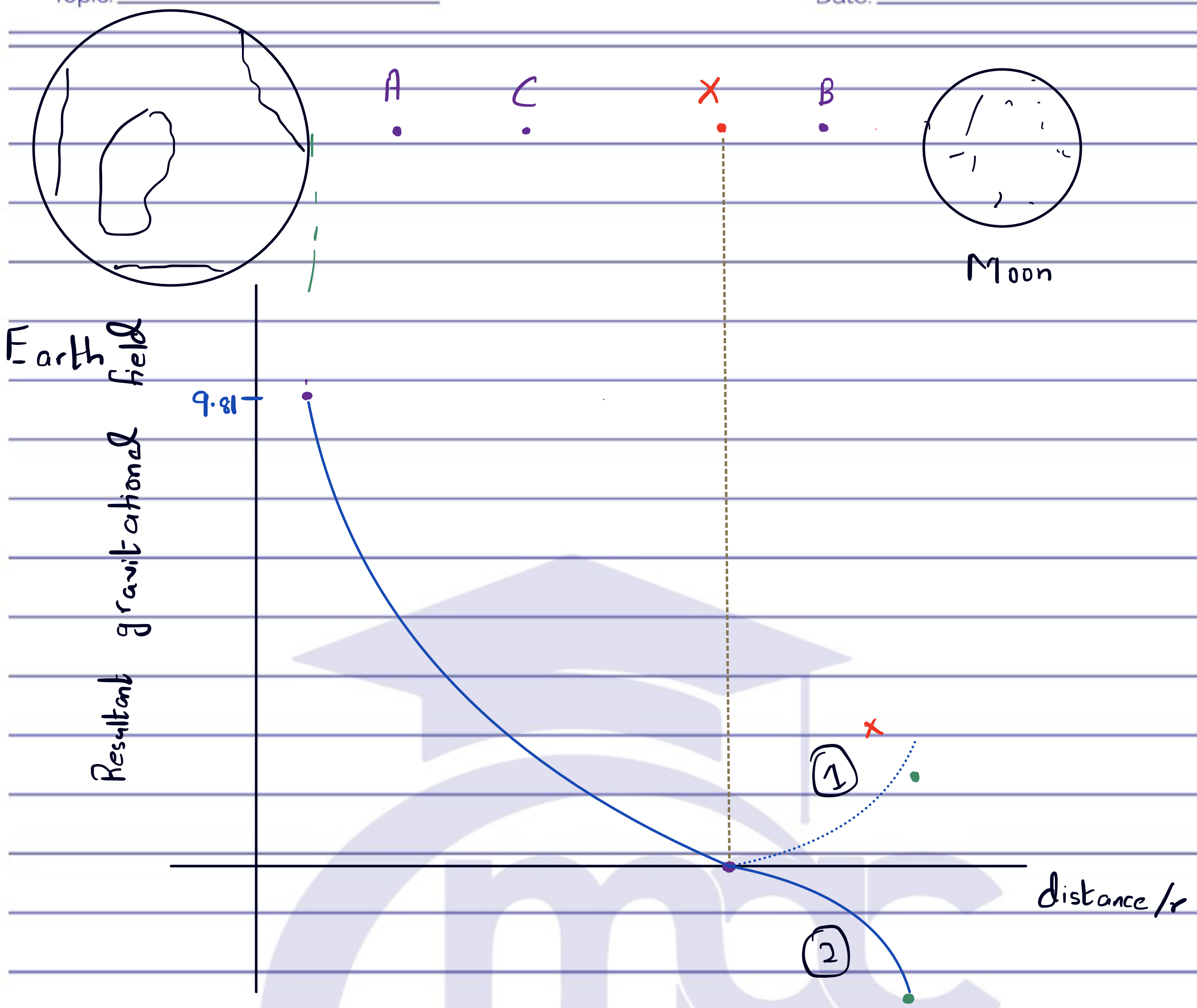

* Gravitational Field strength is a vector quantity.

g(vector) ∴ direction counts hence option (2) preffered over option 1 { opp quadrant }

$\frac{F_g}{} = g$

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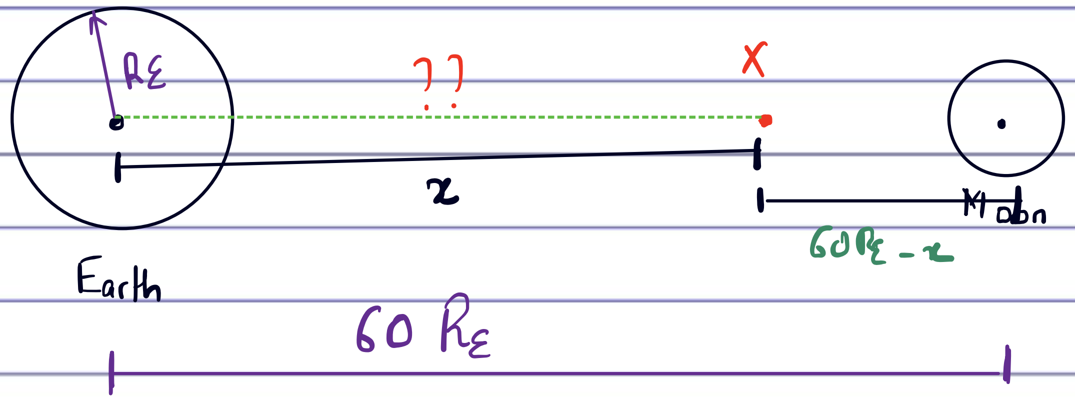
How to mathematically obtain position of the null point.



$g_E = g_m$

If mass of $M_E = 81 M_m$.
in terms of R_E

$$g = \frac{GM}{r^2}$$

$$g_m = \frac{G\ M_m}{(60R_E - x)^2}$$

$$g_E = \frac{G\ M_E}{x^2}$$

$$g_E = \frac{G\ (81 M_m)}{x^2}$$

$$g_m = \frac{G\ M_m}{(60R_E - x)^2}$$

$x = 54\ R_E$

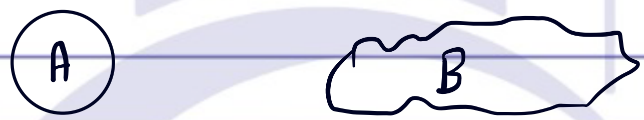
Ex1 Two equal masses but of different radiee state / construct gravitational field lines for each of these masses.

Based on $g = \frac{GM}{R^2}$, $g \propto \frac{1}{R^2}$

if $R \downarrow$; g will $\uparrow$ hence more field lines in B

Ex② Two identical masses are shown below

STate 1 similarities and 1 difference b/w the fields



Similarity : In Both cases field lines will point inwards due to attractive nature.

Difference : In A the gravitational field strength is uniform at every point on the surface but at B the gravitational field strength varies. in strength at all points on the surface owing to the difference in value of R (Due to non uniform shape)

Ex3

- How to calculate gravitational field strength at any pt on the surface of the planet.

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It is given that $\rho_E = 5\rho_x$ $R_E = 2R_x$

mass of source

$$g = \frac{GM}{r^2}$$

$$\rho = \frac{M}{V}$$

$$\frac{4}{3}\pi r^3 \times \rho_E$$

$$\boxed{\frac{\frac{4}{3}\pi G \rho_E R_E^3}{R_E^2}}$$

eq1

$$g_E = \frac{4}{3}\pi G \rho_E R_E$$

$$g = \frac{GM}{r^2}$$

$$g_x = \frac{4}{3}\pi G \rho_x R_x \quad \text{eq 2}$$

$$\frac{g_E}{g_x} = \frac{\cancel{\frac{4}{3}\pi} \cancel{G} \rho_E R_E}{\cancel{\frac{4}{3}\pi} \cancel{G} \rho_x R_x}$$

$$\frac{10}{g_x} = \frac{5}{1} \times \frac{2}{1}$$

$$10 = 10 g_x$$

$$\cancel{\frac{10}{10}} \quad g_x$$

$$\boxed{1\ ms^{-2} = g_x}$$

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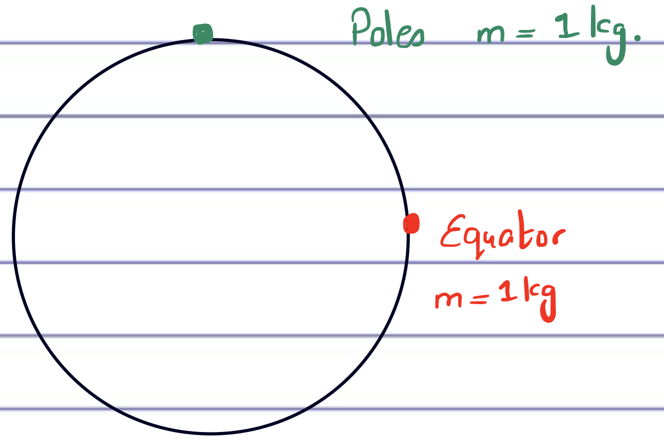
Reasons for variation on the gravitation on the equator vs the poles of the earth.



① Earth is not a perfect sphere Radius vary, g = vary

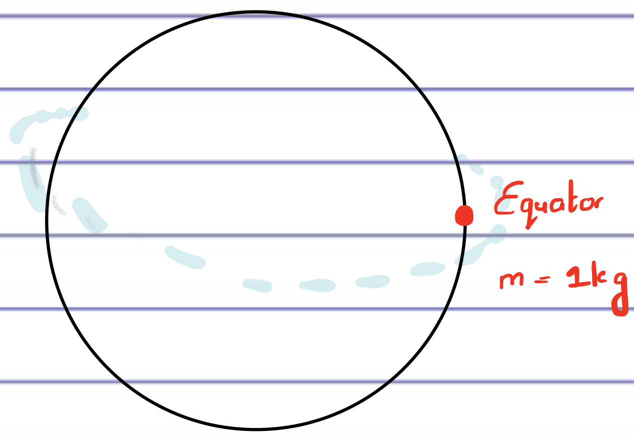
② Non Uniform distribution of Mass in the core of the earth.

$$g = \frac{G\boxed{M}}{R^2}$$ → Distribution is non uniform

Vary (non uniform)

③ Influency of any planetary body eg moon

④ Concept of Circular Motion also has a role to play in making the value of g vary at the Equator & at the poles. This is shown below



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Calculate the F_G for a mass of 1kg positioned at the equator

$$F_G = \frac{GMm}{R^2} = \frac{(6.67\times10^{-11})(6\times10^{24})(1)}{(6.4\times10^{6})^2}$$

$$F_G = 9.77N$$

Calculate the centripetal force whith the mass required in order to perform circular motion at the equator.

$$F_c = mR\omega^2$$

$$F_c = (1)(6.4\times10^{6})\left(\frac{2\pi}{T}\right)^2$$

$$T = 24\times60\times60$$

$$= 0.034N$$

* Since gravitational force provides centripetal force For Circular motion to take place ∴ out of 9.77N, 0.034N is Used up in making particle perform circular motion hence centripetal Force avalible at the equator will only be

$$R_F = 9.77 - 0.034$$

$$= 9.736$$

Now based on $F = mg$

$$9.736 = 1(g)$$

$$g = 9.736\ m/s^2$$

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Calculate the F_G b/w Earth and mass positioned at the poles?

$$F_G = \frac{GMm}{R^2} = \frac{(6.67\times10^{-11})(6\times10^{24})(1)}{(6.4\times10^{6})^2}$$

9.77.

Cal F_c required by the object to perform circular motion at the poles.

The object at the pole is not performing circular motion rather only spinning about its own axis hence $F_c(\text{pole}) = 0$

Resultant Force $= 9.77$

$F = mg$

$9.77 = 1\times g$

$\boxed{9.77 = g}$

Conclution: Variation in g occurs because centripetal Acceleration is required at the Equator to perform circular motion; whereas there is no such requirement at the polar regions.

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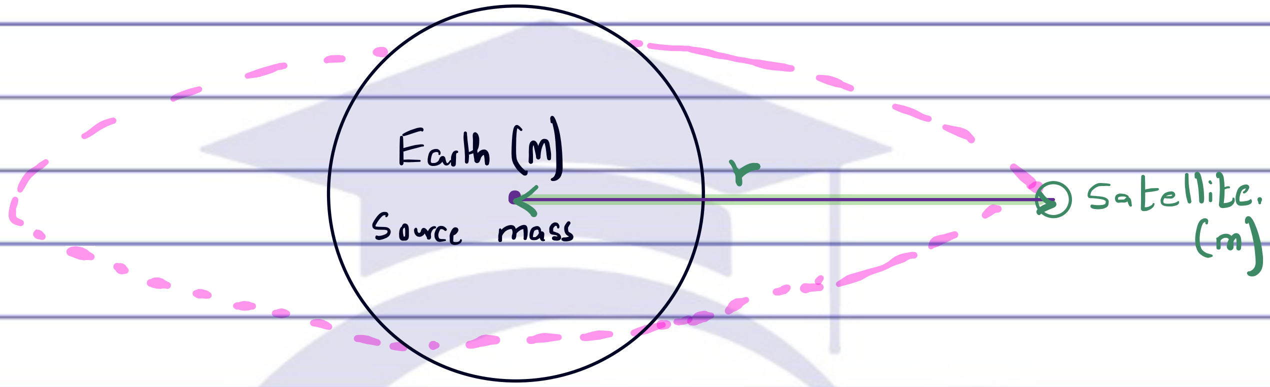
Expression for obtaining
i) linear velocity
ii) Angular velocity
iii) Kinetic Energy
iv) Time period for a satellite performing circular motion. around Earth.



"r" = denotes distance b/w satellite and and Centre of the earth

OR

r denotes radius of circular motion. of satellite.

(i) linear velocity.

— For CM to occur, it is the gravitational force F_G which provides the centripetal force. hence

$$F_G = F_C$$

$$\frac{GMm}{r^2} = \frac{mv^2}{r} \Rightarrow \boxed{v = \sqrt{\frac{GM}{r}}}$$

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ii) Angular velocity. (ω)

$$F_G = F_c$$

$$\frac{GMm}{r^2} = mr\omega^2 \Rightarrow \omega = \sqrt{\frac{GM}{r^3}}$$

Since GM is a constant so we can conclude that v and ω both depend on the value of "r"

iii) Kinetic Energy of the satellite.

$$KE = \frac{1}{2}mv^2$$

$$KE = \frac{1}{2}m\left(\sqrt{\frac{GM}{r}}\right)^2$$

$$\boxed{\Rightarrow KE = \frac{GMm}{2r}}$$

* We have established $v = \sqrt{\frac{GM}{r}}$ So replace this result in KE formula.

iv) Time Period

$$T = \frac{2\pi}{\omega} \Rightarrow \frac{2\pi}{\sqrt{\frac{GM}{r^3}}} \Rightarrow \boxed{T = 2\pi\sqrt{\frac{r^3}{GM}}}$$

OR

$$\boxed{T^2 = \frac{4\pi^2}{GM}r^3}$$

* We have already established that $\omega = \sqrt{\frac{GM}{r^3}}$

Since $\frac{4\pi^2}{GM}$ is constant

$$\boxed{T^2 \propto r^3}$$ Kepler's law of Planetary Motion.

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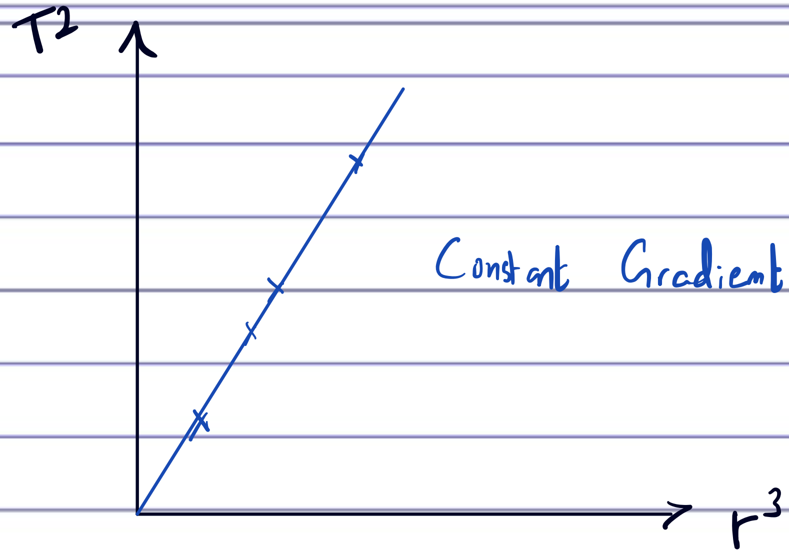
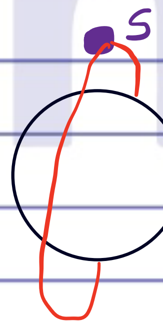


Constant Gradient.

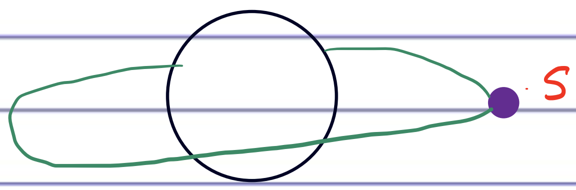
→ Gravitation lecture 3

Types of Satellite

① Polar orbiting Satellite.



② Equitorial orbiting satellites.



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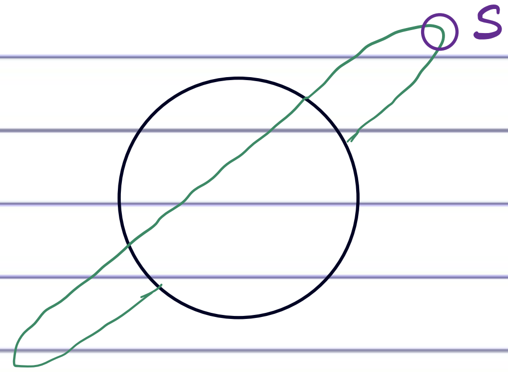
③ GPS (Global positioning Satellite)



They can simultaneously monitor both the polar regions & the equitorial regions.

Equitorial orbiting Satellites are further classified into Two types.

① Geo Stationary satellites (Synchronous satellite) *Imp

② Non Geo Stationary or (A Synchronous satellite)

* A satellite is said to be geostationary if it appears remains stationary when viewed from Earth's surface.

* For any satellite to be classified as Geo Stationary it must satisfy the following condition

① Time period of satellite must be equal to time period of the earth.

② Angular velocity (ω) of the satellite must be equal to the angular velocity of the earth since $\omega = \frac{2\pi}{T}$

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(3) Earth spins about its Equator as its axis ∴ the satellite must also be launced in an equitorial orbit.

(4) Earth spin about its axis in the direction West to East ∴ the satellite must also have the same sense of rotational i.e from west to East.

Radius of a Geostationary Orbit.

Q) At what distance (r) must a satellite be launched in order for it to be classified as Geostationary

$$T^2 = \frac{4\pi^2}{GM} \times r^3$$

$$(24 \times 60 \times 60)^2 = \frac{4\pi^2}{(6.67 \times 10^{-11})(6 \times 10^{24})} r^3$$

=

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$$G = 6.67 \times 10^{-11}$$
$$M = 6.0 \times 10^{24}$$
$$R_E = 6.4 \times 10^{6}$$

$$(24 \times 60 \times 60)^2 = \frac{4\pi^2}{(6.67 \times 10^{-11})(6 \times 10^{24})} r^3$$

Solve for r

$$\boxed{r = 4.2 \times 10^{7}\ m}$$

$$\frac{4.2 \times 10^{7}}{6.4 \times 10^{6}} \approx 6.6 R_E$$

Note: Radius of Geo stationary orbit is approx $6.6 R_E$ (6.6 times the Radius of Earth)

Conclusion: Satellite can be lauched at any distance in any plane, but if they launched in an equitorial orbit at a distance of $4.2 \times 10^7 m$ away from the earth's centre only Then they can be classified as Geo stationary satellites.

What is the purpose of Geo stationary satelites.

Since they appear to remain stationary above any point on the Earth's surface they are generally used for

* long term weather forcasting
* long term communication link. (Sports eg cricket, football)

Can we use them for "Spying"

No they cannot be used for spying because if they remain stationary they can be easily "Tracked".

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