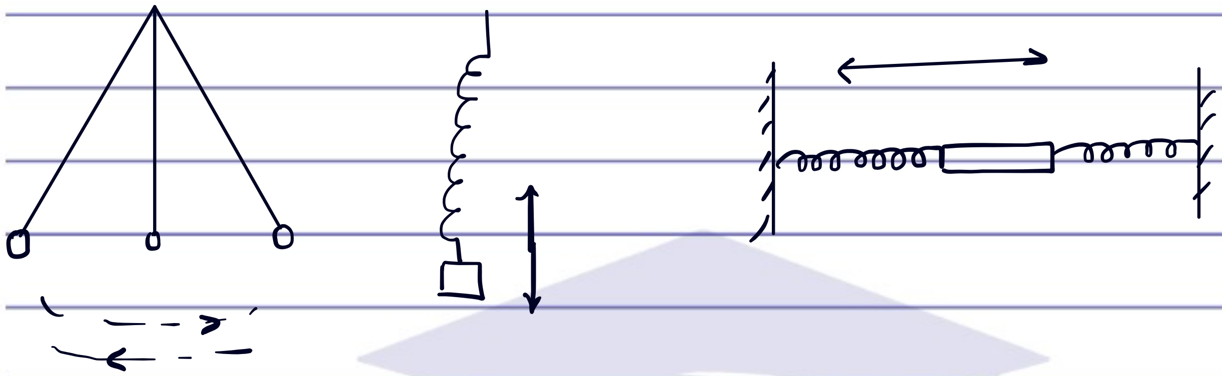
Topic: Oscillation [Simple harmonic motion] Date:

Basic Idea about the topic:

→ In Physics oscillations or oscillatory motion is called simple harmonic motion.

Examples.



V, S, a, t, F & Energy of oscillating body.

Simple Harmonic motion.

A periodic motion (repetative motive) in which acceleration is directly propotional to the negative of displacment.

* A restoring force acts towards the mean position all the time during oscillation.

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Basic Idea of differenciation:

① $y = 2\sin x$ $\frac{dy}{dx} = 2\cos x$	② $y = \sin 3x$ $\frac{dy}{dx} = 2\cos 3x \times 3$ $= 6\cos 3x$	③ $x = x_0 \sin \omega t$ $x = x_0 \sin \omega t$ $\frac{dx}{dt} = x_0 \omega \cos \omega t$
④ $y = 3\cos x$ $\frac{dy}{dx} = -3\sin x$	⑤ $y = 4\cos 2x$ $\frac{dy}{dx} = -4\sin 2x \times 2$ $= -8\sin 2x$	⑥ $y = 4\cos 2x.$ $\frac{dx}{dt} = -x_0 \sin \omega t$ $= -x_0 \omega \sin \omega$

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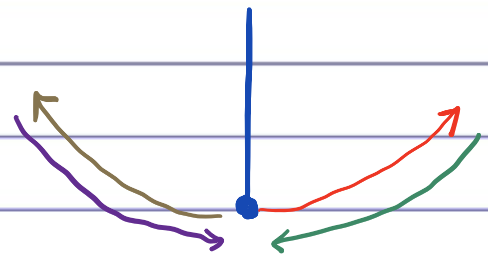
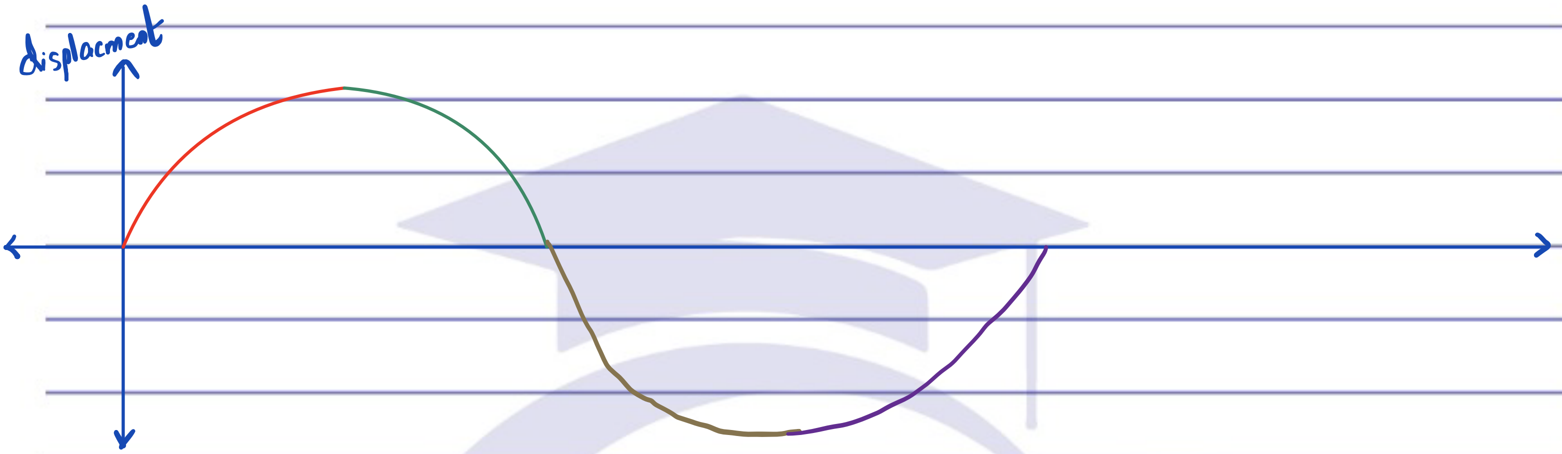
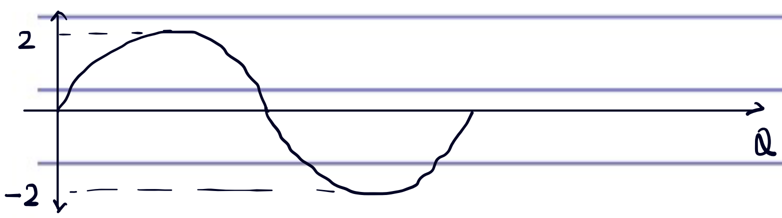
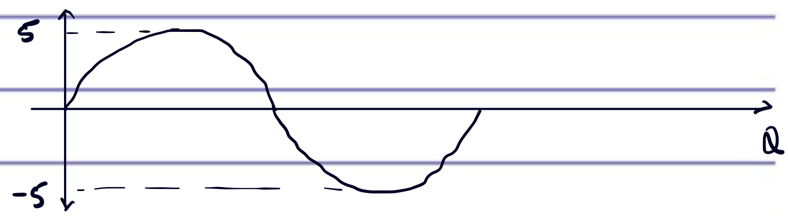
Graphical representation for a particle for a particle performing Simple harmonic motion.





Q) The graph below shows how displacment (x) varies with (θ)





Equation of this above graph = $2 \sin \theta$

Equation of this above graph = $5 \sin \theta$

Amplitude.

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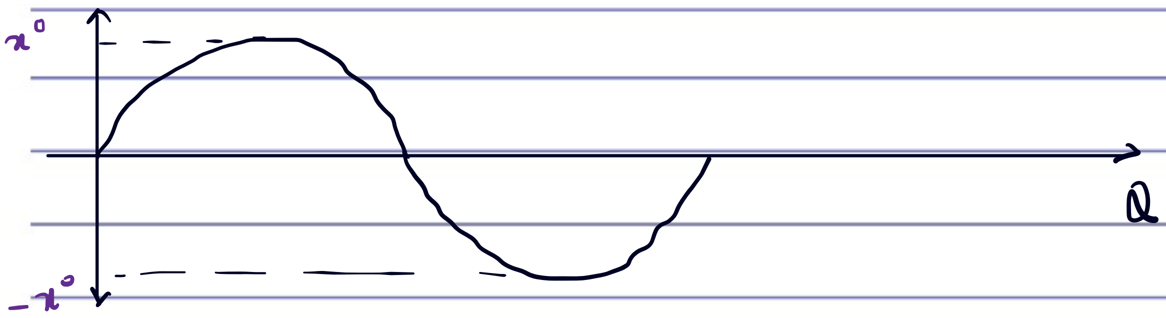

Equation of this above graph $= -x_0 \sin\theta$

$x = x_0 \sin\theta$ $\qquad\qquad \omega = \dfrac{\theta}{t}$

$\boxed{x = x_0 \sin \omega t}$ — (1) $\qquad\qquad \omega t = \theta$

The above equation represents a function of displacement (x) against time (t) for any particle which perform S.H.M.

x = displacment $\qquad t$ = time

x_0 = amplitude. $\qquad \omega$ = angular velocity (Circular motion)

ω = angular frequency. (Simple Harmonic Motion)

$x = x_0 \sin\omega t$ — (1)

The $\dfrac{dx}{dt}$ of displacment gives us velocity.

$$\frac{dx}{dt} = x_0 (\cos\omega t) \times \omega$$

$$\boxed{v = x_0 \omega \cos(\omega t)} \text{ — (2)}$$

$\dfrac{dv}{dt}$ gives us acceleration.

$$\frac{dv}{dt} = x_0 \omega \cos(\omega t)$$

$$= x_0 \omega (-\sin\omega t)\,\omega$$

$$\boxed{= -x_0 \omega^2 \sin\omega t} \text{ — (3)}$$

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From eq 1 and 3 we can conclude.

$a = -x_0 \omega^2 \sin \omega t$ $\qquad x = x_0 \sin \omega t$

$\boxed{a = -x\omega^2}$ —— (4)

For any particle to perform S.H.M it must satisfy the equation

$$a = -\omega^2 x$$

a = Acceleration
x = displacement
ω = angular Frequency.

2 more results based of Eq 4 are.

(1) Acceleration is directly proportional to displacement.
(2) Acceleration and displacement are both oppositly directed. (-ve sign)

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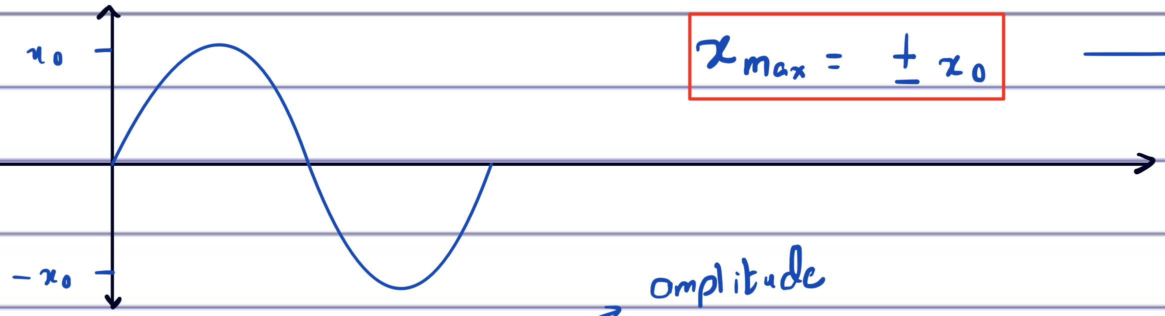
Q) Construct graphs of

① displacement (x) vs t $\quad x = x_0 \sin \omega t$

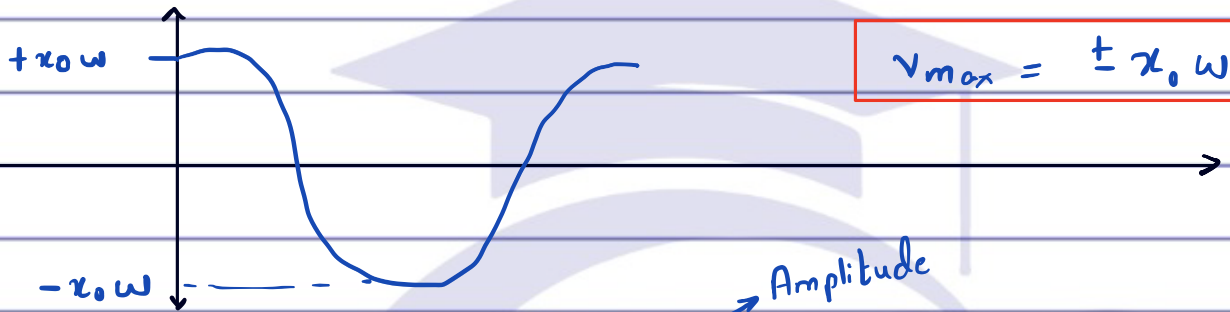
amplitude: x_0; shape: $\sin \omega t$



$\boxed{x_{max} = \pm x_0}$ — ⑤

② Velocity (v) vs t $\quad v = x_0\omega \cos \omega t$

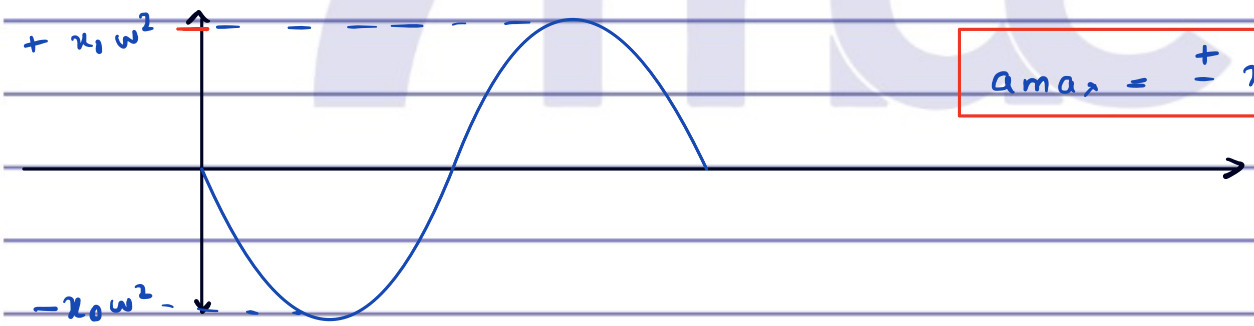
amplitude: $x_0\omega$; shape: $\cos \omega t$.



$\boxed{v_{max} = \pm x_0 \omega}$ — ⑥

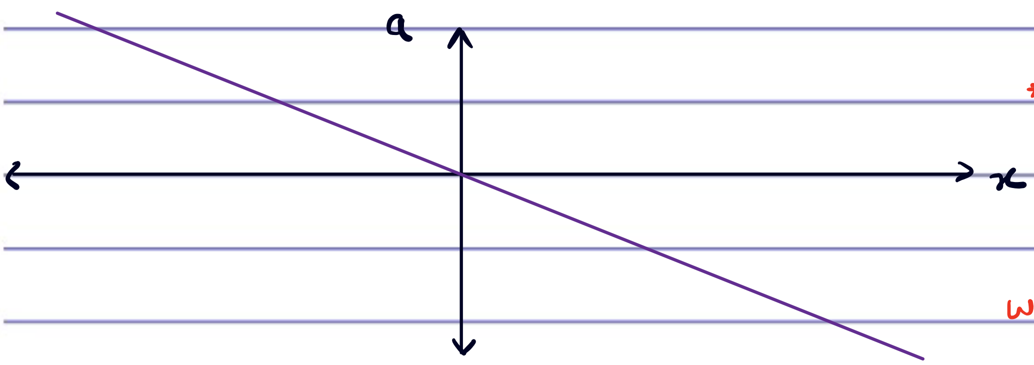
③ acc (a) vs t $\quad a = -x_0\omega^2 \sin \omega t$

Amplitude: $x_0\omega^2$; Shape: $\sin \omega t$.



$\boxed{a_{max} = \pm x_0 \omega^2}$ — ⑦

④ acc (a) vs x $\quad a = -\omega^2 x$



$y = kx$

* Straight line passing through origin with a negative gradient where gradient $= -\omega^2$

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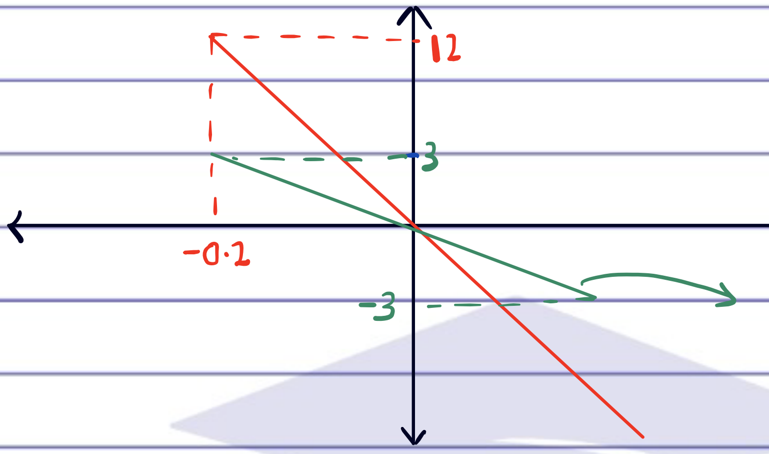
Q) For the graph and values given below
(i) Calculate T of oscillation.
(ii) Suggest new shape of the graph if frequency of oscillation is halved.



gradient of new graph becomes 1/4th

(i) gradient $= -\omega^2$

$$-60 = -\left(\frac{2\pi}{T}\right)^2$$

$$T = 0.81\,s$$

ii) new shape of the graph

$$a = -\omega^2 x$$

$$a = -\left(\frac{2\pi}{T}\right)^2 x \qquad T = \frac{1}{f}$$

$$a = -(2\pi f)^2 x$$

$$a = -4\pi^2 f^2 x$$

$$a = -4\pi\left(\frac{1}{2}f\right)^2 x$$

$$a = \frac{-\cancel{4}\pi f^2 x}{\cancel{4}} \Rightarrow -\pi f^2 x$$

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Uphill now

$x = x_0 \sin \omega t$ $x_{max} = \pm x_0$

$v = x_0 \omega \cos \omega t$ $v_{max} = \pm x_0 \omega$

$a = -x_0 \omega^2 \sin \omega t$ $a_{max} = \pm x_0 \omega^2$

$a = -\omega^2 x$

Expression / Formula relating the velocity of a particle performing SHM with its displacement. (x) No need for Derivation.

$$\boxed{v = \pm \omega \sqrt{x_0^2 - x^2}} \quad (8)$$

ω = angular frequency

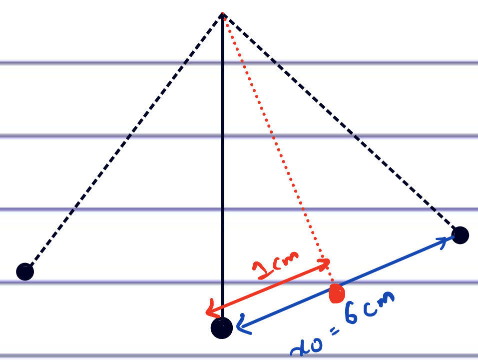
x_0 = amplitude

x = displacement of particle at any given instant

v = velocity of the particle at that

$T = 2s$

$x_0 = 6cm$



Calculate the velocity of the particle at the instant when its displacement is 1cm

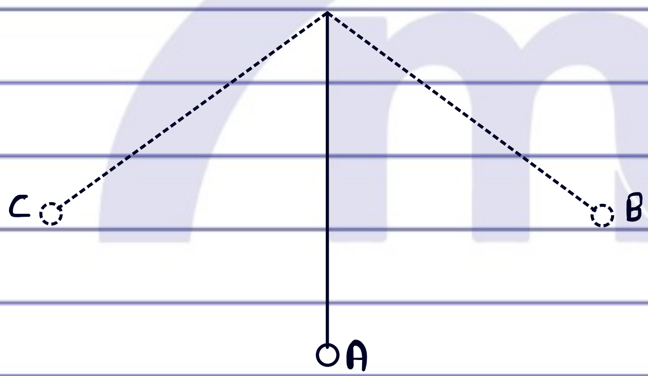
$v = \pm \omega \sqrt{x_0^2 - x^2}$

$v = \pm \frac{2\pi}{2} \sqrt{(0.08)^2 - (0.01)^2}$

$v = \pm$

this $\pm$ indicates that oscillating particle will cross each point twice during one complete cycle in opposite direction hence the two signs.

How to compare displacement, velocity and Acceleration at different pts in an oscillatory motion.



At A (mean position)

i) disp = 0

ii) Velocity = maximum

$\boxed{V_{max} = \pm x_0 \omega}$

iii) Since in SHM

$a \propto x$ if $x = 0$

a = zero

At B or C (Extreme Pts)

i) disp = maximum

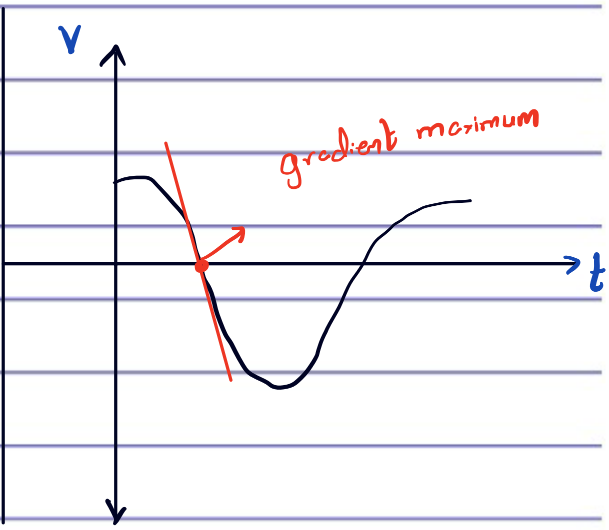
ii) will come to rest. at that instant $v = 0$

iii) Since $a \propto x$

if x is max then

a is maximum

$\boxed{a_{max} = \pm x_0 \omega^2}$



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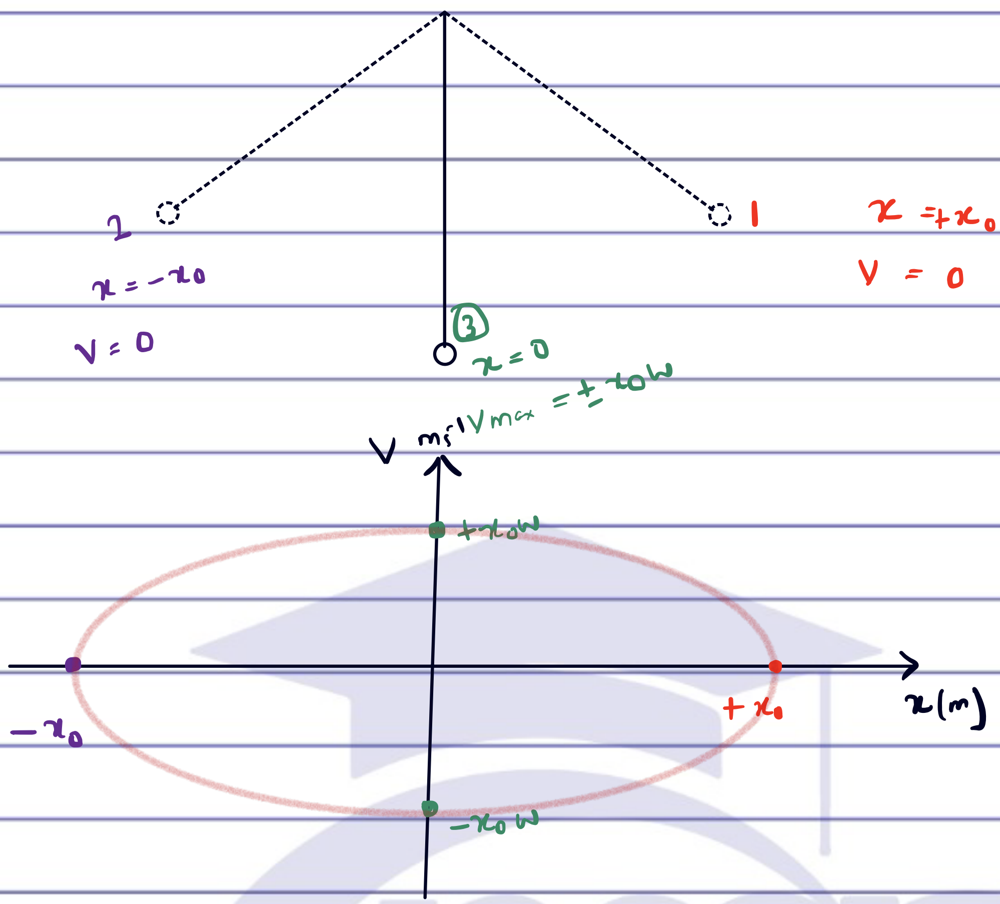
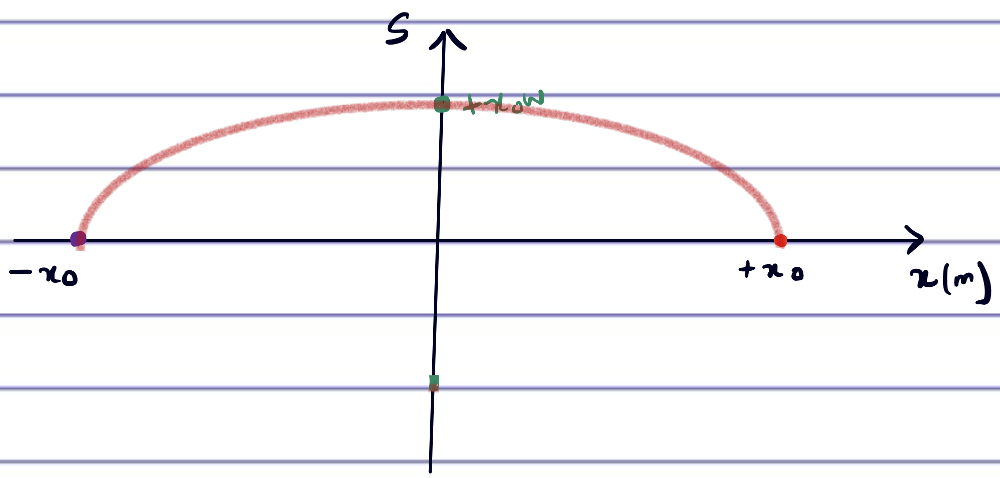
Graph of Velocity (v) against displacement (x)



Graph of Speed (s) against displacement (x)



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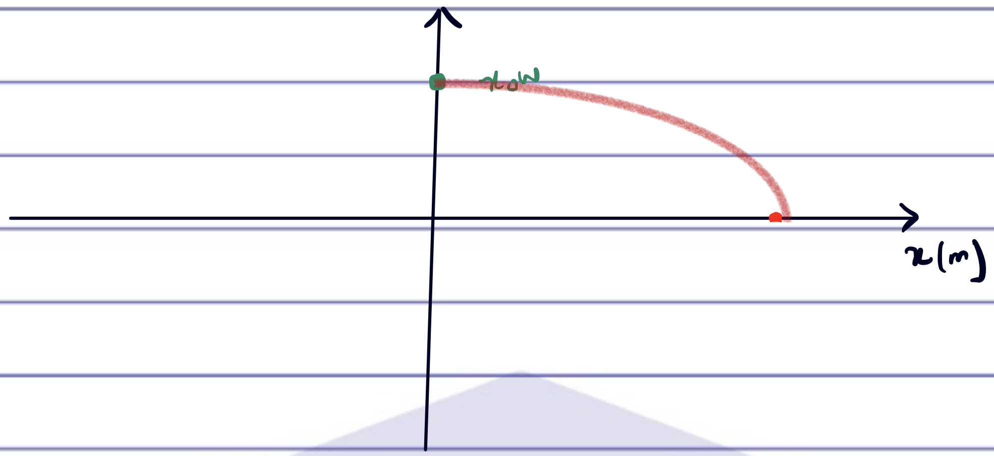
Graph of Speed (s) against distance (d)



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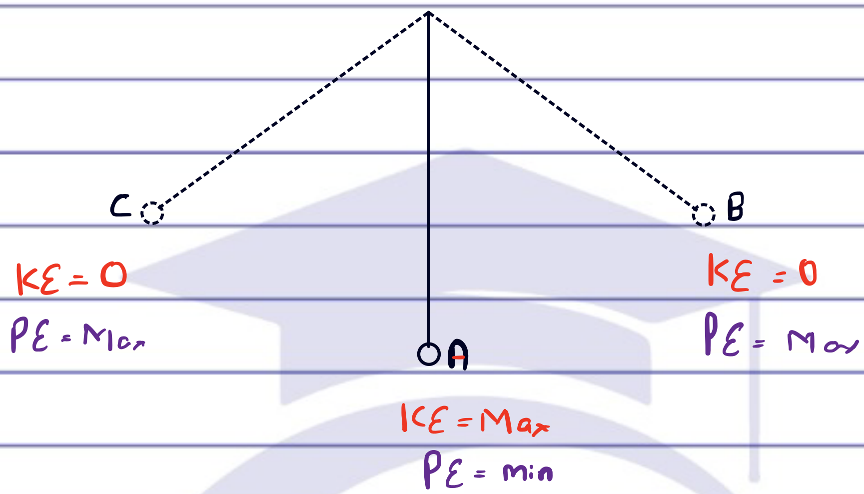
Formulas for Calculating KE, PE and TE for a particle of mass m performin S.HM.

* Based on law of conservation of Energy, for an oscillating particle its KE can interchange into its potential energy but total Energy at Every point will remain constant



$$\boxed{KE\,(Max) = \frac{1}{2} m\omega^2 x_0^2}$$ No need for derivation. —— (9)

$$\boxed{KE\,(\text{General formula}) = \frac{1}{2} m\omega^2 (x_0^2 - x^2)}$$ —— (10)

$$\boxed{PE_{max} = \frac{1}{2} m\omega^2 x_0^2}$$ —— (11)

$$\boxed{PE\,(\text{general formula}) = \frac{1}{2} m\omega^2 x^2}$$ —— (12)

Since T.E is constant only 1 equation

$$\boxed{TE = \frac{1}{2} m\omega^2 x_0^2}$$ —— (13)

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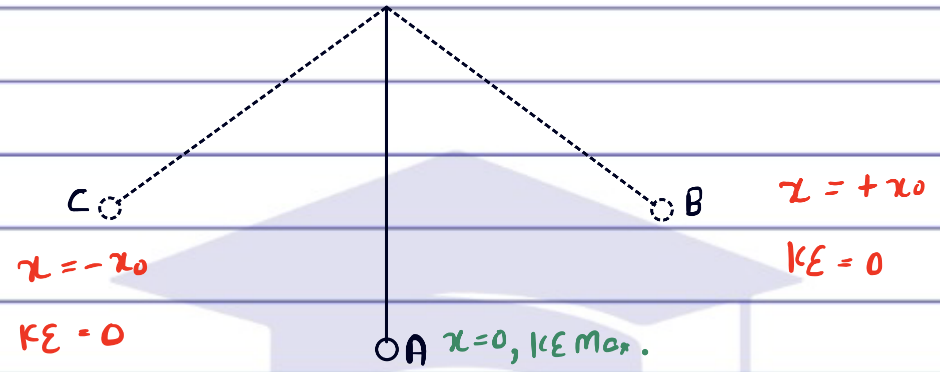
ω = angular freq

m = mass

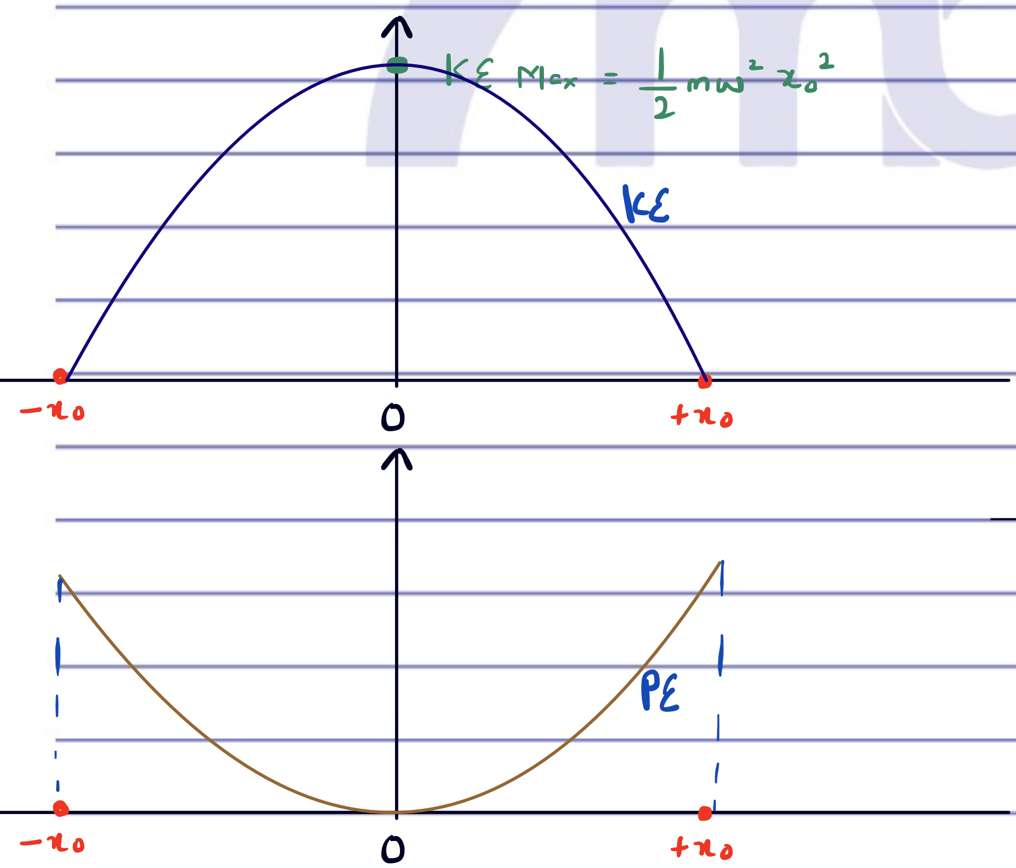
x_0 = amplitude of oscillation

x = displacement at any given instant.

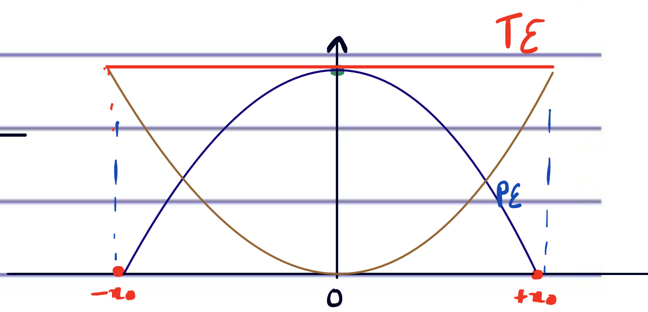
Graphs of K.E, PE & TE as a function of displacement x for a particle performing S.H.M



⇒ The mean position is taken to be the reference pt for the graph shown below.



Combined graph.



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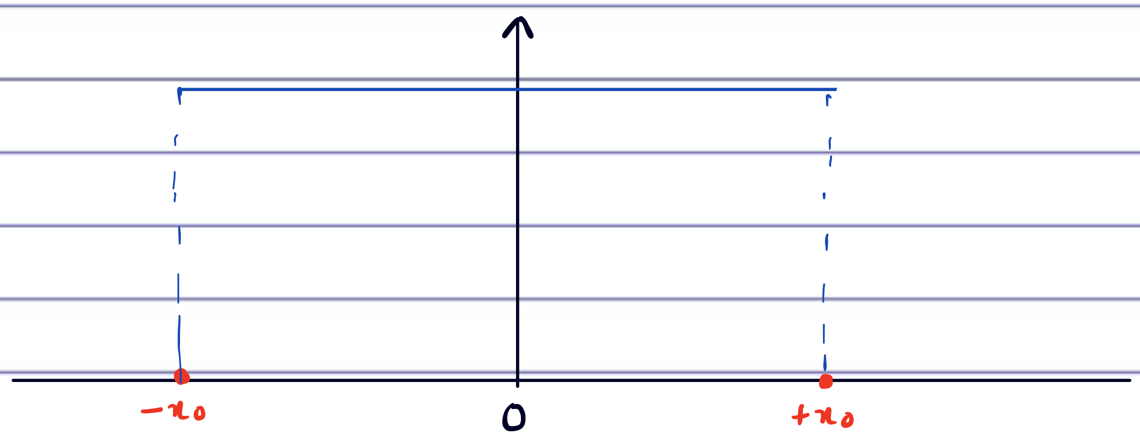
Since T.E = KE + PE = constant

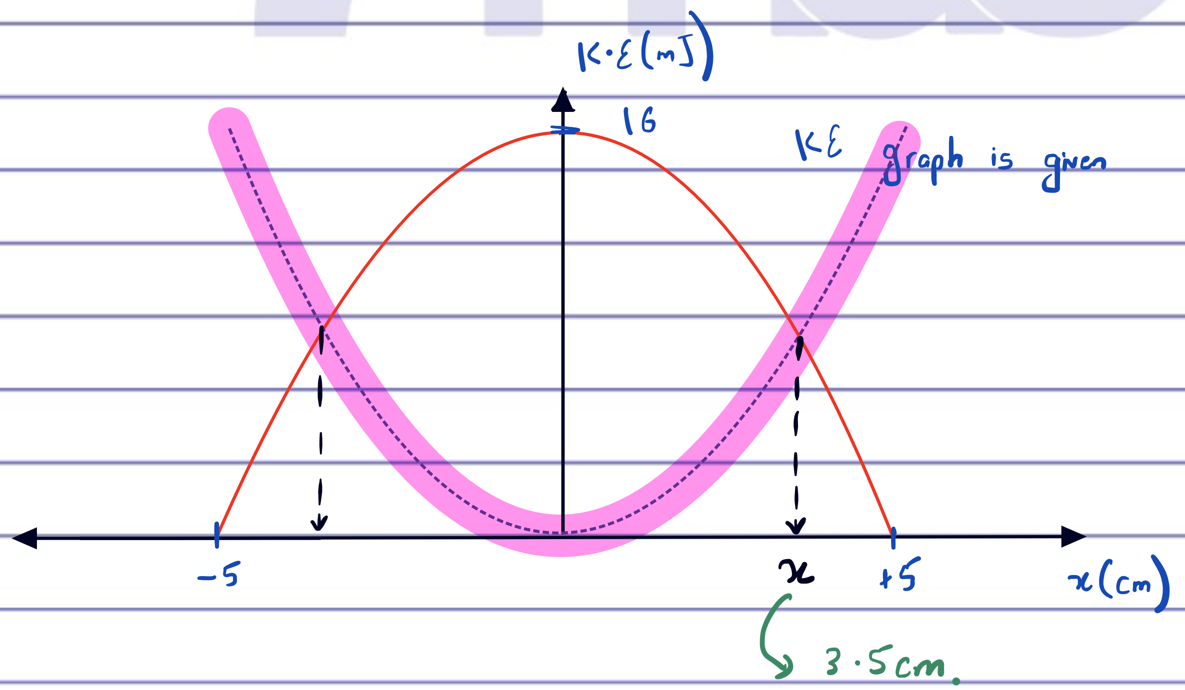


Questions for graph given below

i) Calculate the angular frequency "ω" of the particle whose m = 70g

ii) Sketch graph of PE vs x on the same Axis.

iii) Calculate the displacment (x) when the particle has equal amounts of KE & PE.



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Formula # 9

i) $K.E_{max} = \frac{1}{2} m\omega^2 x_0^2$

$16 \times 10^{-3} = \frac{1}{2}(0.070)\,\omega^2\,(0.05)^2$

$\boxed{\omega = 13.5}$

iii) disp x when $KE = PE$

KE (general form) = PE (General form)

$\cancel{\frac{1}{2}}\,\cancel{m}\cancel{\omega^2}\,(x_0^2 - x^2) = \cancel{\frac{1}{2}}\,\cancel{m}\cancel{\omega^2}\,x^2$

$x_0^2 - x^2 = x^2$

$x_0^2 = 2x^2$

$5^2 = 2x^2$

$\sqrt{\frac{25}{2}} = x$

$\sqrt{12.5} = x$

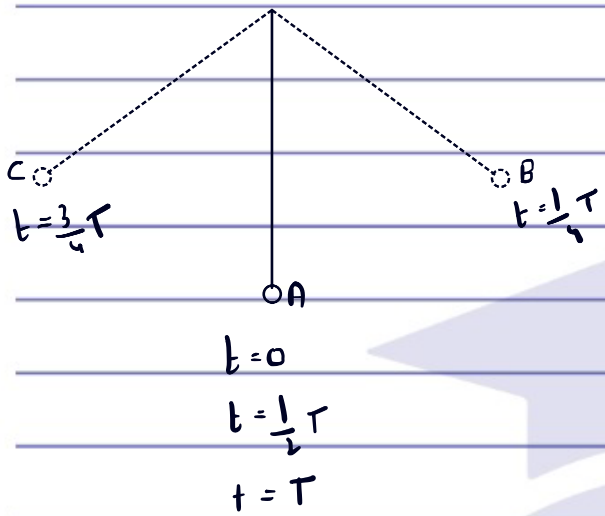
$x = 3.5\text{ cm}$

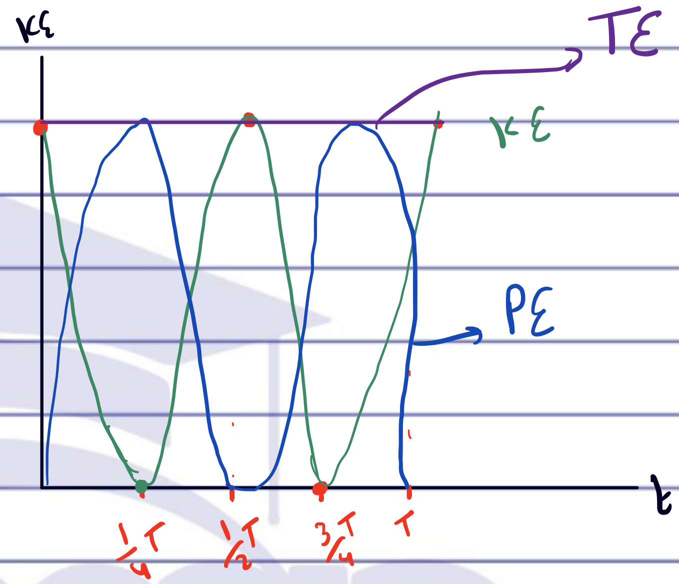
$\boxed{x = \frac{x_0}{\sqrt{2}}}$

Note: The above equation indicates if amplitude x_0 is divided by $\sqrt{2}$ the answer gives us the disp x at which particle's KE and PE are both equal.

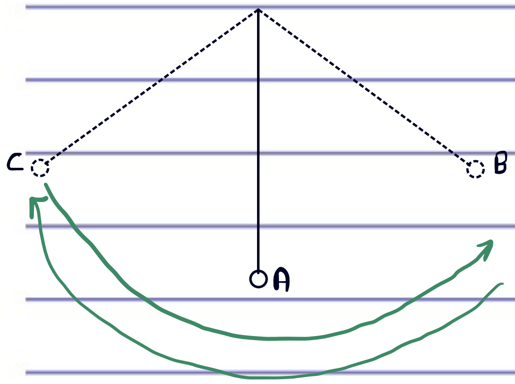
* Graphs of KE, PE and Total Energy against time (t) for a particle performing S.H.M.

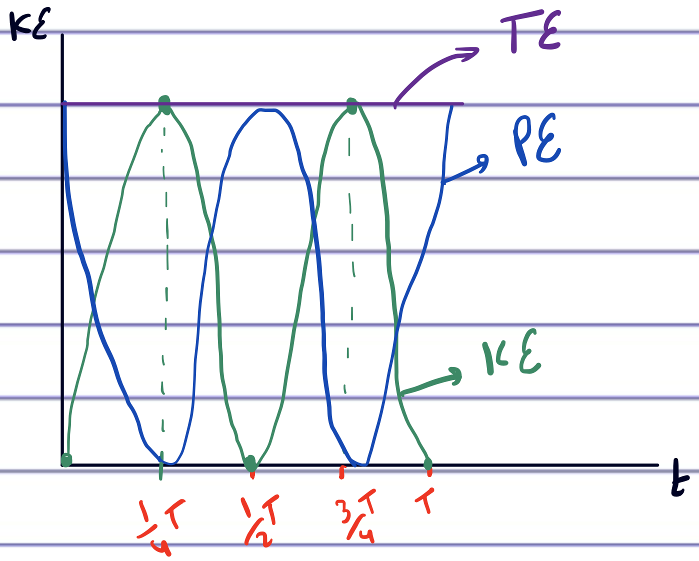
eg 1 : Lets assume that mean position ($x=0$) is the reference point





eg 2 lets assume any one of the extreme position is taken as the reference point.





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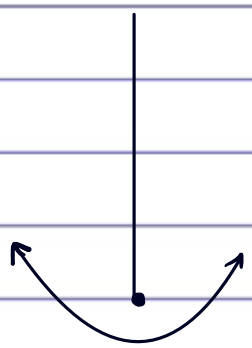
Derivation Not required.

1. Formula for calculating time period and frequency.

For a pendulum performing oscillations.



$$\boxed{T = 2\pi\sqrt{\frac{l}{g}}} \quad (14)$$

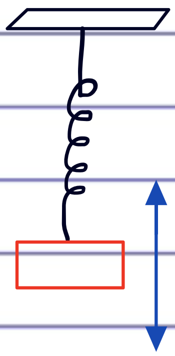
Inverse of T gives f

l = length of pendulum.

g = acceleration of freefall (9.81)

$$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}} \quad (15)$$

2. Formulas for calculating time period and frequency for a mass attached to a spring oscillating in vertical plane.



$$\boxed{T = 2\pi\sqrt{\frac{m}{k}}} \quad (16)$$

OR

$$\boxed{T = 2\pi\sqrt{\frac{e}{g}}} \quad (17)$$

$$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}} \quad (18)$$

OR

$$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{g}{e}}} \quad (19)$$

m = mass of oscillating Particle

g = acc due to gravity

e = extention of the spring

K = Spring constant of the spring

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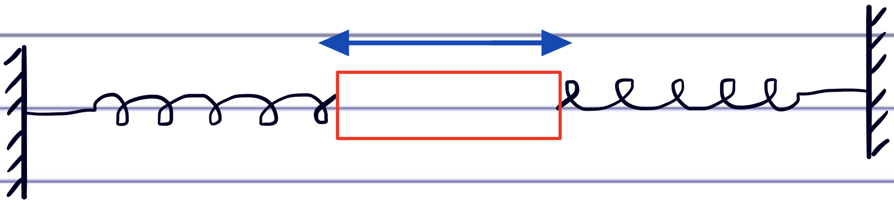
(3) Formulas for Calculating time period and frequency for a mass attached with 2 identical springs. & made to oscillate in a horizontal plane.



$$\boxed{T = 2\pi\sqrt{\frac{m}{2k}}} \quad (20)$$

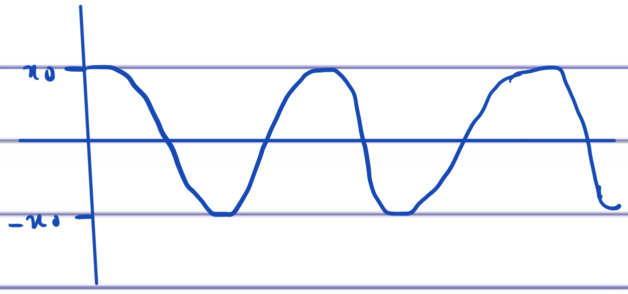
2k because there are 2 springs.

$$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{2k}{m}}} \quad (21)$$

Topic: Types of Oscillation Date:

① Free oscillations.

Oscillations which takes place in absence of any Resistive medium, for eg Pendulum oscillating in vaccum.



* amplitude x_0 remains constant
* Time Period, angular frequency and frequency remains constant.

→ Damped oscillation

* Refers to oscillations that takes place in some kind of resistive medium.
* Depending on the amount of Resistance offered by any medium damped oscillation can be further subdivided into 3 categories.

① light damping → In Air
② Criticle damping (Medium damping) → In Water.
③ Heavy damping → In Mercury, ketchup.

→ ② & ③ are not valid for A2 level. {b/c due to viscosity it will come to rest before 1 oscillation} {Hence No SHM}

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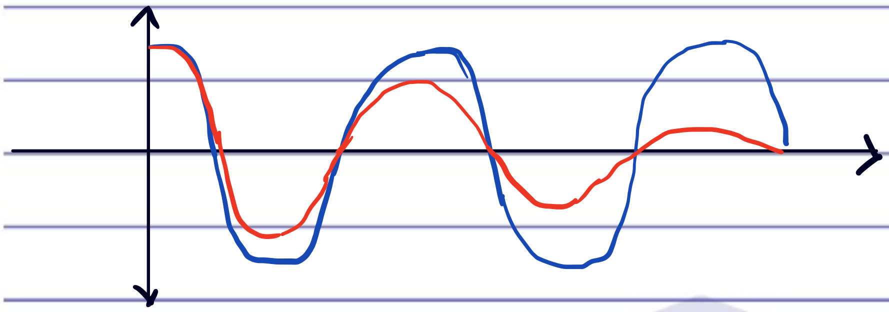
For a pendulum oscillating in Air (i-e example of light Damping)

* Following Properties can be established.



In light Damping.

* Amplitude & TE decreases exponentially / Non uniformally.

* Timeperiod, ω (angular freq) and frequency remains constant.

Q1) In light damping it can be observed that amplitude reduces from 1cm to 0.8cm in the first 10 oscillations.

Can we say that in next 10 oscillation the amplitude will drop from 0.8cm to 0.6cm.

No because based on property of light damping, the amplitude decreases exponentially / Non linearly

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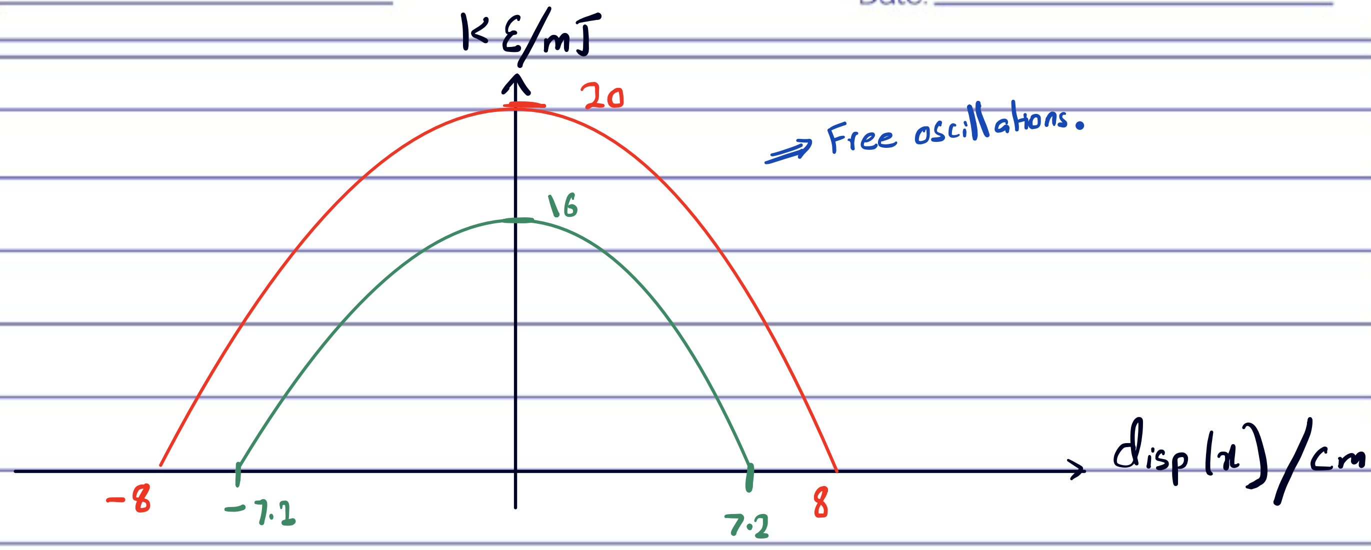
Q2)



* graph is of free oscillation.
* light damping causes the energy to reduce by 20%

Sketch the new graph given on the same axis.

$\frac{80}{100} \times 20 = 16 \text{ mJ}$

$\text{Max KE} = \frac{1}{2} m\omega^2 x_0^2$

Find the new amplitude $x_0 = ?$

$20 = \cancel{\frac{1}{2}} \cancel{m} \cancel{\omega^2} x_0^2$ —— 1

$16 = \cancel{\frac{1}{2}} \cancel{m} \cancel{\omega^2} (x_{0_{new}})^2$ —— 2

$\frac{20}{16} = \frac{x_0^2}{x_{0_n}^2} \Rightarrow \frac{20}{16} = \frac{64}{x_{0_{new}}}$

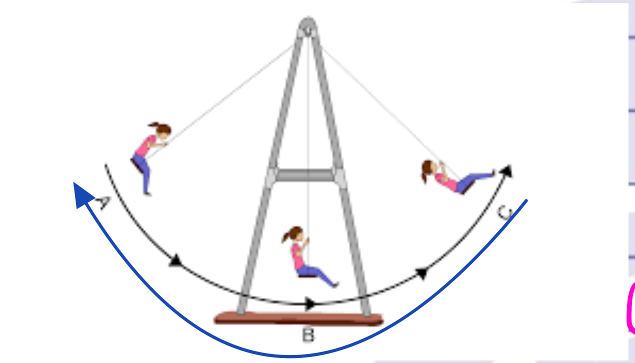
$\boxed{x_{0_{new}} = 7.2}$

Topic: Resonance is SHM. Date:

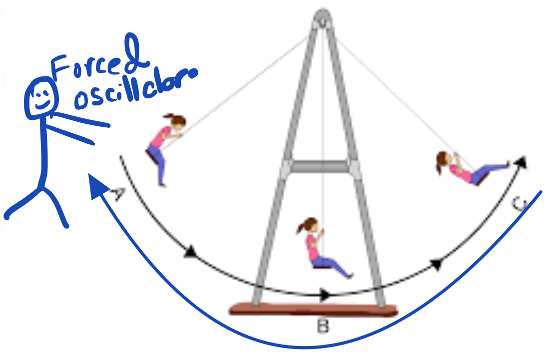
Every mechanical system has its own frequency of oscillation which is called the natural Frequency or fundemental frequency It is denoted by symbol of "f_o"

Experiments have shown that if a forced oscillator is made to vibrate close to this mechanical system. A transfer of Energy takes place from the forced oscillator to the mechanical system. causing amplitude of mechanical system to increase.

Forced oscillator: An external oscillator which applies periodic force on a mechanical system.



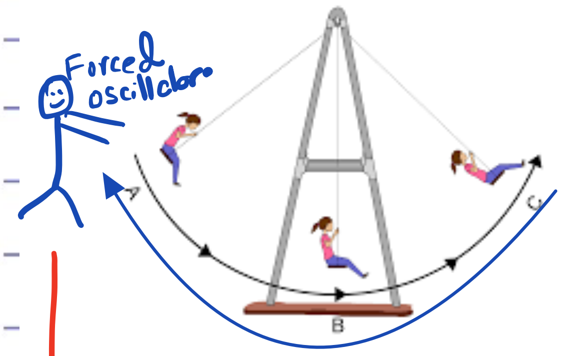
If child is Swinging on its own at 5 Hz freq this 5 Hz is the natural frequency. denoted by $(f_o) = 5\,Hz$.



Amplitude will increase.

When the frequency of the forced oscillator becomes compatible or Equal to the Fundemental frequency $(f_o = 5\,Hz)$ the amplitude of vibration of the mechanical system is known to reach its maximum value this phenomena is called as Resonance

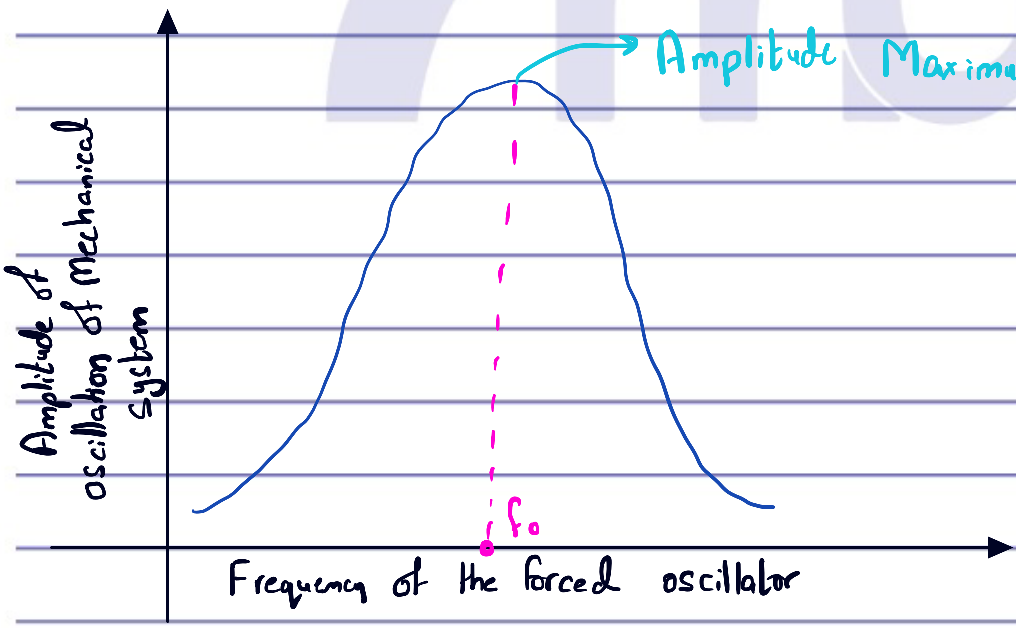
Freq of forced oscillator = f_o (Fundemental freq)



If forced oscillator is providing 5Hz

* Maximum transfer of Energy as both Systems are in phase with Each other. Amp of mechanical System to reach its maximum value called Resonance.

Graph of Resonance



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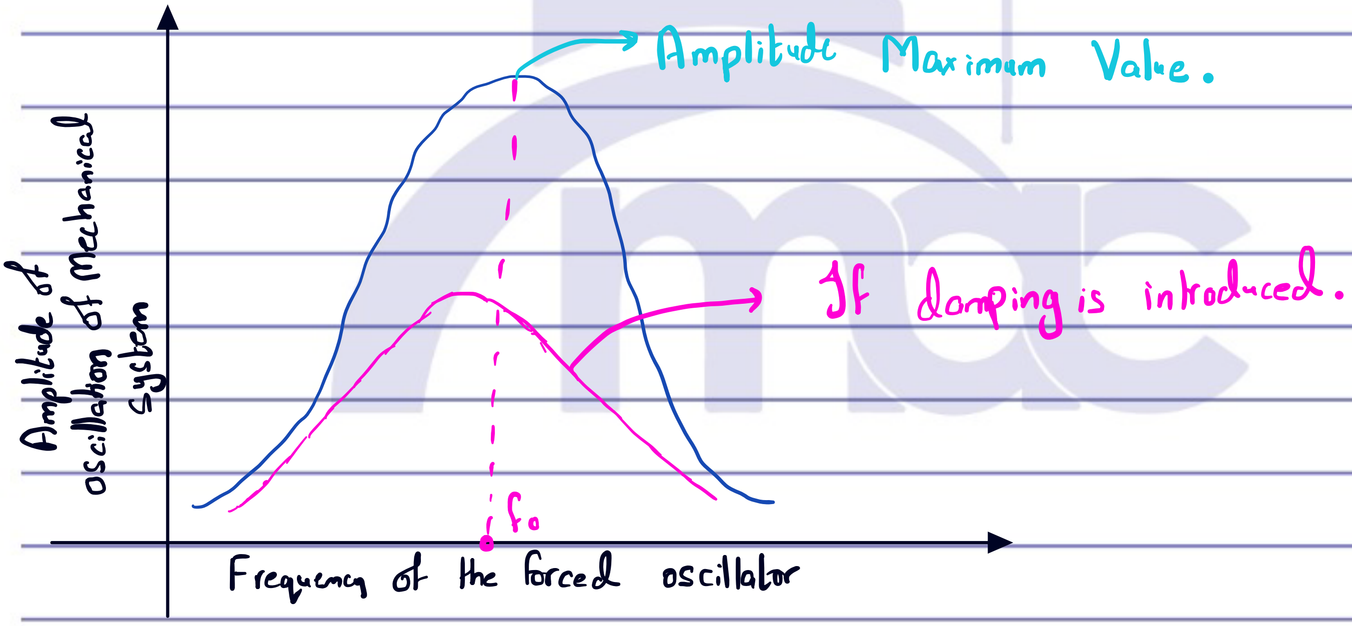
Sketch on the same Axis a new graph assuming that the new system is now subjected to light damping?

Ans)

* The spread of graph will decrease (less
* Amplitude will reduce.
* Peak Flattens (f_0 shifts slightly)

$$\downarrow E = \frac{1}{2} m \omega^2 \downarrow x_0^2 \downarrow$$

$2\pi f \downarrow$



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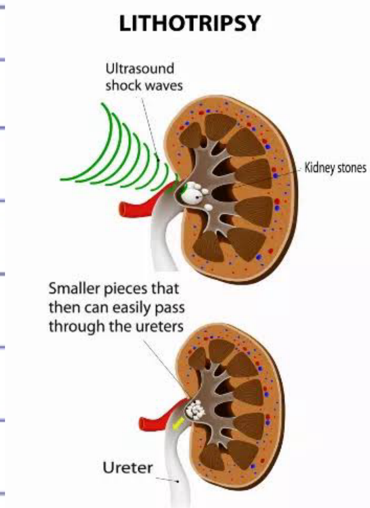
⇒ Usefulness of Resonance.

- device lithotripter.

This device is known to produce shockwaves to break kidney stones.



- In Microwaves oven.

⇒ Microwaves produced in Oven have a specific frequency which matches the freq of water molecules present in food. Hence when microwaves are directed towards food particles, the water molecules vibrate and food is cooked / warmed.

- Idea of Resonance is also used in MRI.

⇒ Where should Resonance be avoided.

- Opera Singer (glass break)

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